Determining Modern Energy Functional for Nuclei And The Status of The Equation of State of Nuclear Matter

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INTRODUCTION

Nuclear physics:

Study of structure, interactions and properties of nuclei. Aim to quantitatively understand and relate vast amount of properties of nuclei in terms of few constituents, elementary laws and processes.

Current situation:

Active area of research. We have a certain picture (understanding) obtained through 70 years of phenomenological research, qualitative consideration and application of laws quantum mechanics. Q. M. is very successful in describing properties of nuclei. There is no evidence contradicting Q. M.

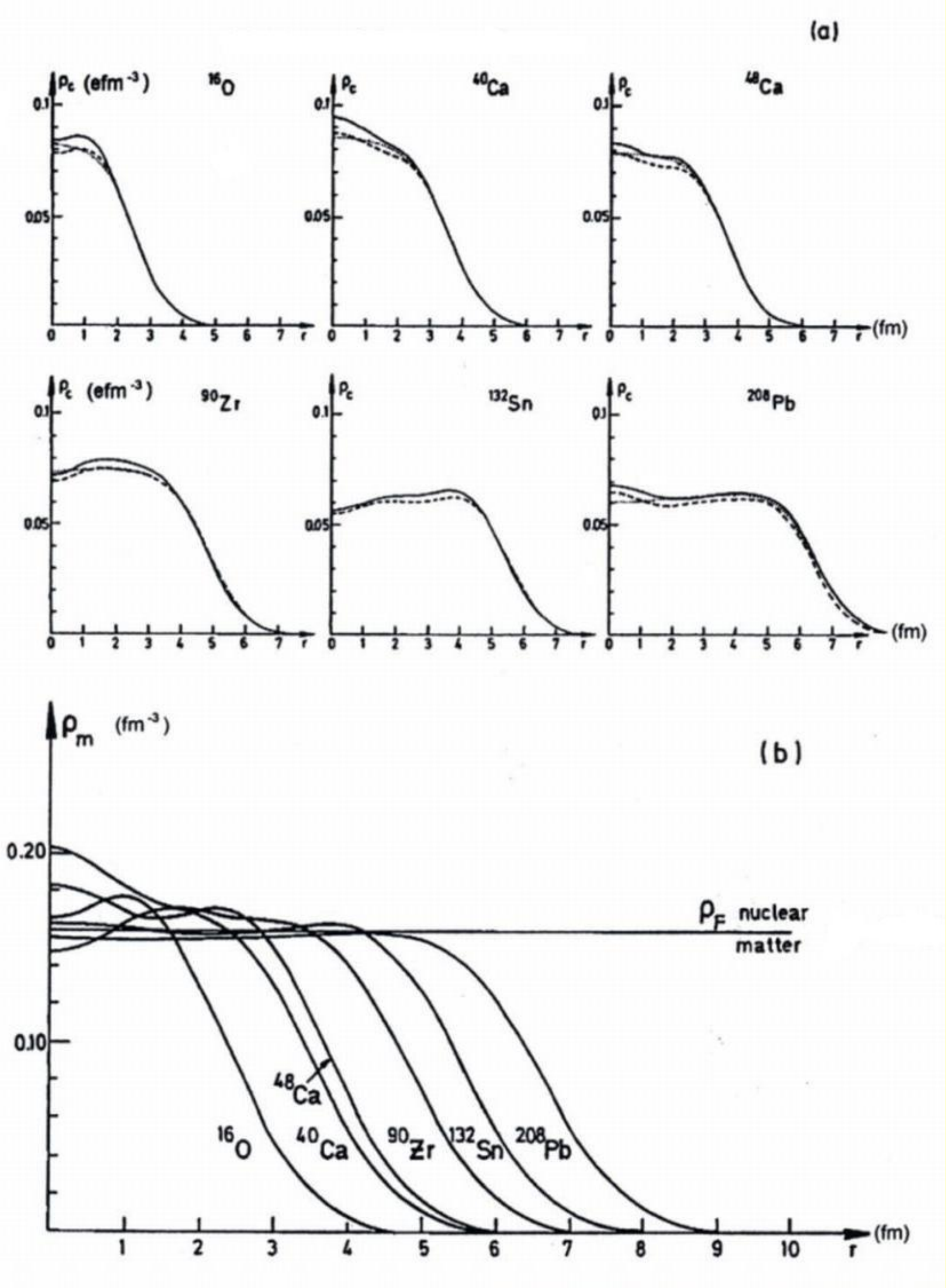
Recent emphasize:

Properties of nuclei under extreme conditions of excitation energy (temperature), angular momentum and N-Z (asymmetry).

Relation to other areas:

Astrophysics: source of energy stars, structure and evolution of stars, origin of the elements, neutron stars and supernova.
Other systems: atomic clusters, metal clusters, trapped ions, and mesoscopic systems.

Matter and charge density distributions



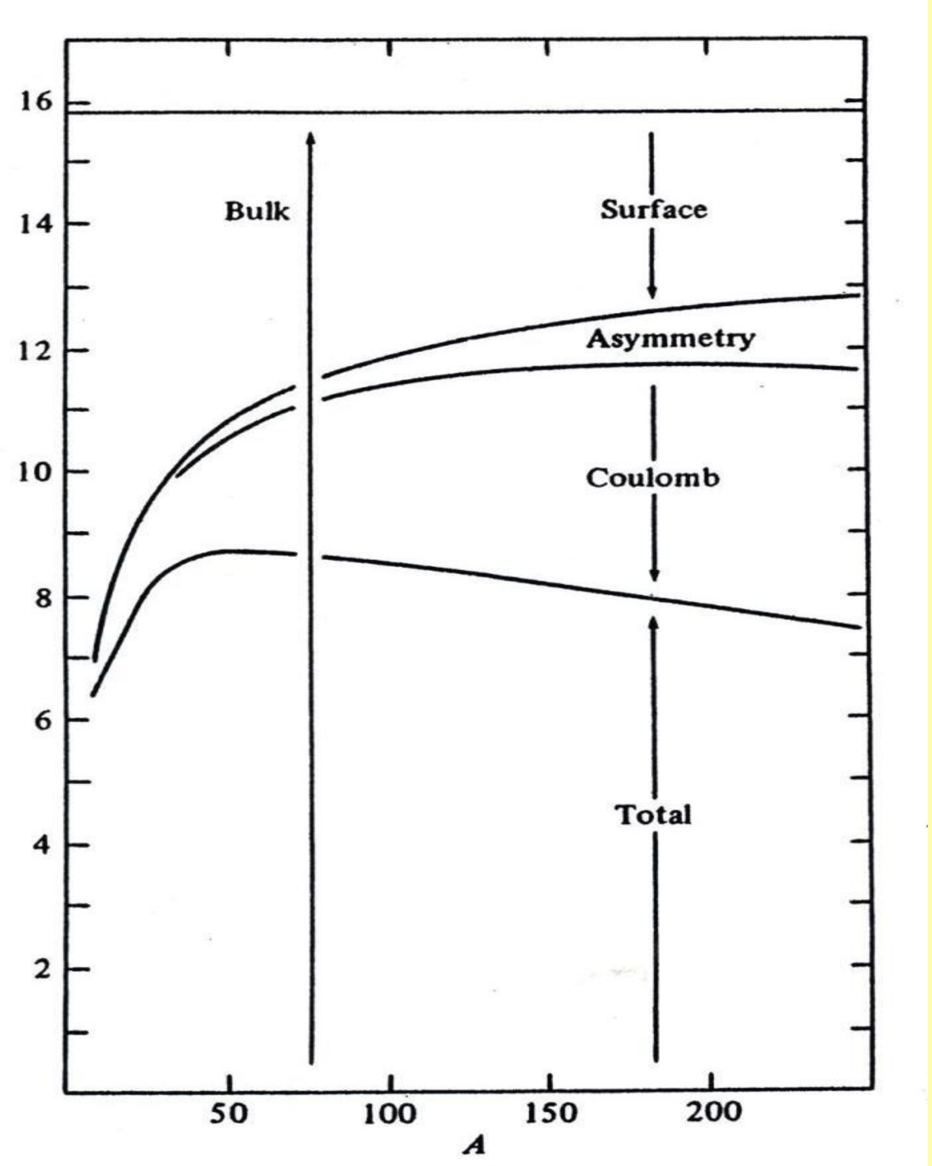
(a) Charge density distribution $\rho_c(r)$ for doubly –magic nuclei ^{16}O, ^{40}Ca, ^{90}Zr, ^{132}Sn, and ^{208}Pb. The theoretical curves are compared with the experimental data points (units are ρ_c(efm^{-3}) and r(fm)).

(b) Nuclear matter density distributions ρ_m(fm^{-3}) for the magic nuclei.

$$\rho(r) = \frac{\rho_0}{1 + e^{\frac{r - R_0}{a}}}$$

$$R_0 = r_0 A^{1/3} \qquad r_0 \approx 1.1\, fm$$

$$a = 0.55\, fm$$



The contribution to B/A. Note that the surface, asymmetry and Coulomb terms all subtract from the bulk term.

Nuclear Binding (Weizsacker formula)

$$BE(Z,N) = a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(N-Z)^2}{A} + \frac{\delta}{A^{1/2}}$$

$a_1 = 15.568$ MeV (Volume)

$a_2 = -17.226$ MeV (Surface)

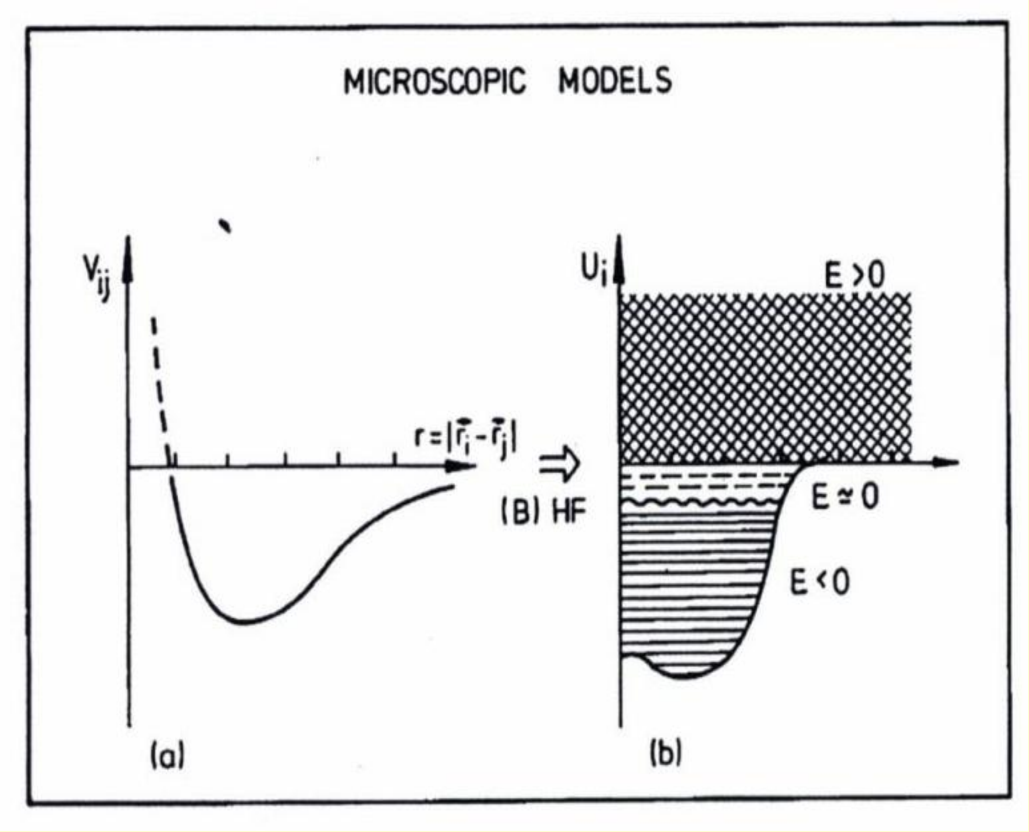
$a_3 = -0.698$ MeV (Coulomb)

$a_4 = -23.7$ MeV (Symmetry)

$\delta =$ -11.2 MeV (*odd-odd*)
0 (*even-odd*) (pairing)
11.2 MeV (*even-even*)

Mean-field approximation

The many-body Schrödinger equation *Hψ = E ψ* is difficult to solve. In the mean-field approximation each particle moves independently from other nucleons in a single particle potential, representing its interactions with all other nucleons.



$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j} V_{ij}$$

Approximation:

$$H = \sum_i \left[\frac{\vec{p}_i^{\,2}}{2m_i} + U(\vec{r}_i) \right] + H_{res}$$

$$H_0 = \sum_i h_i, \qquad h_i = \frac{\vec{p}_i^{\,2}}{2m_i} + U(\vec{r}_i)$$

$$h_i \phi_i = E_i \phi_i$$

$$\Phi = \mathrm{A} \phi_i(1) \ldots \phi_A(A)$$

$\mathrm{A} \equiv$ Antisymmetrization operator (fermions) or symmetrization operator (bosons)

Spherical symmetry

Wood-Saxon potential popular:

$$U(r) = \left[1 - 0.67\frac{N-Z}{A}\tau_Z\right]\left[U_0 f(r) + U_{s.o.}\vec{\sigma}.\vec{l}\ \frac{1}{r}\frac{df(r)}{dr}\right] + \frac{1}{2}(1-\tau_Z)\ U_{Coul}(r)$$

$$f(r) = \left[1 + \exp\left[\frac{r-R}{d}\right]\right]^{-1}$$

$$U_{Coul}(r) = \begin{cases} \dfrac{Ze^2}{2R_c}\left(3 - \left(\dfrac{r}{R_c}\right)^2\right) & r \le R_c \\ \dfrac{Ze^2}{r} & r > R_c \end{cases}$$

$$R = R_c = 1.27 A^{1/3} fm, \quad d = 0.67 fm, \quad U_0 = -51 \text{ MeV}, \quad U_{s.o.} = -0.22 U_0$$

Spherical symmetry $\phi(\vec{r}) = \dfrac{u(r)}{r} Y_{ljm}(\theta,\phi)\chi_\tau$

$\left[-\dfrac{\hbar^2}{2m}\nabla^2 + U(r)\right]\phi(\vec{r}) = \varepsilon\phi(\vec{r})$ is reduced to $-\dfrac{d^2u}{dr^2} + s(r)u(r) = 0$

where $s(r) = \dfrac{2m}{\hbar^2}\left(\varepsilon - U(r) - \dfrac{l(l+1)\hbar^2}{2mr^2}\right)$

The spin-orbit coupling: describe real nuclei

For the harmonic oscillation + spin orbit interaction, the energy eigenvalues become

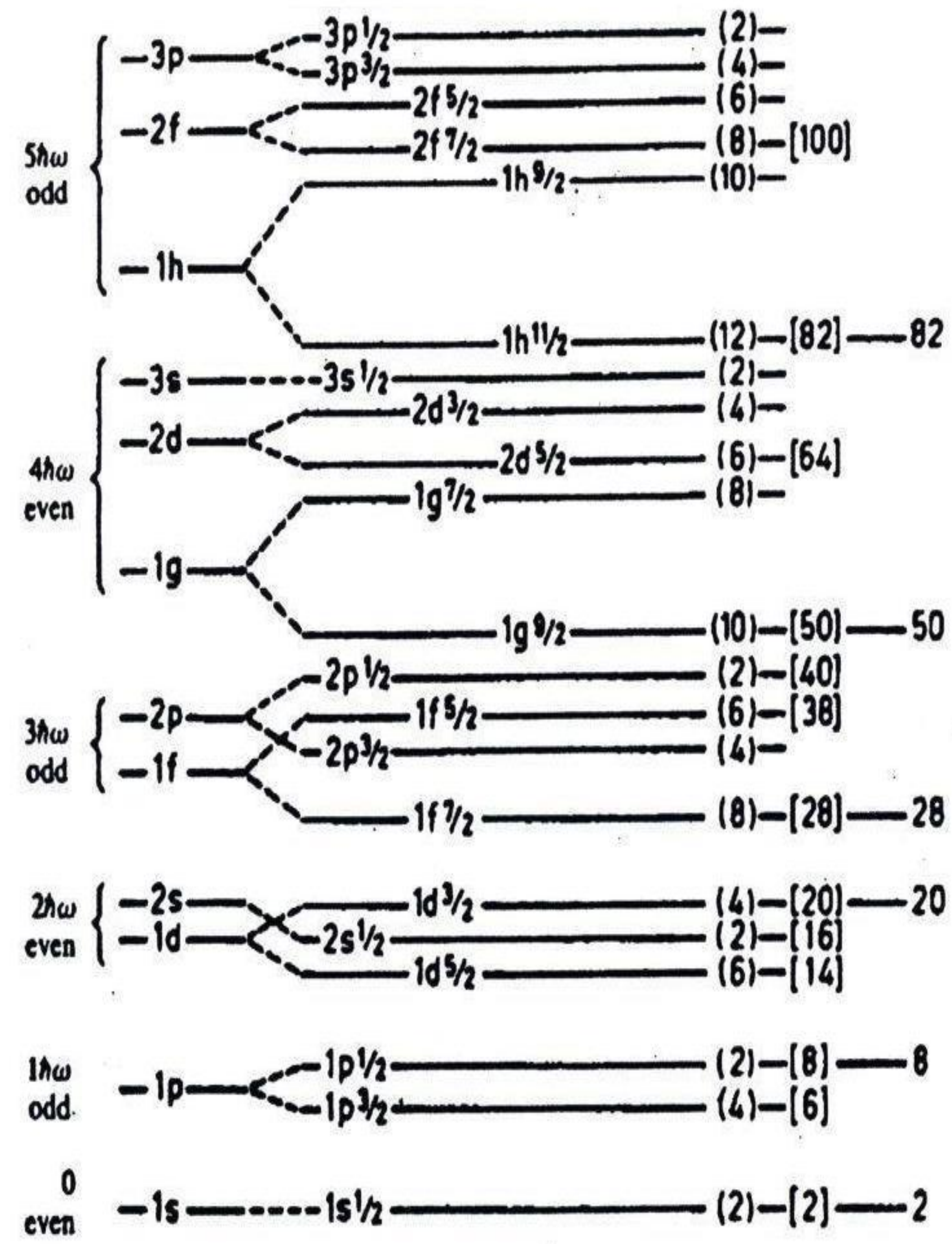
$$\varepsilon_{n(ls)j} = \left[2(n-1) + l + 3/2\right]\hbar\omega$$

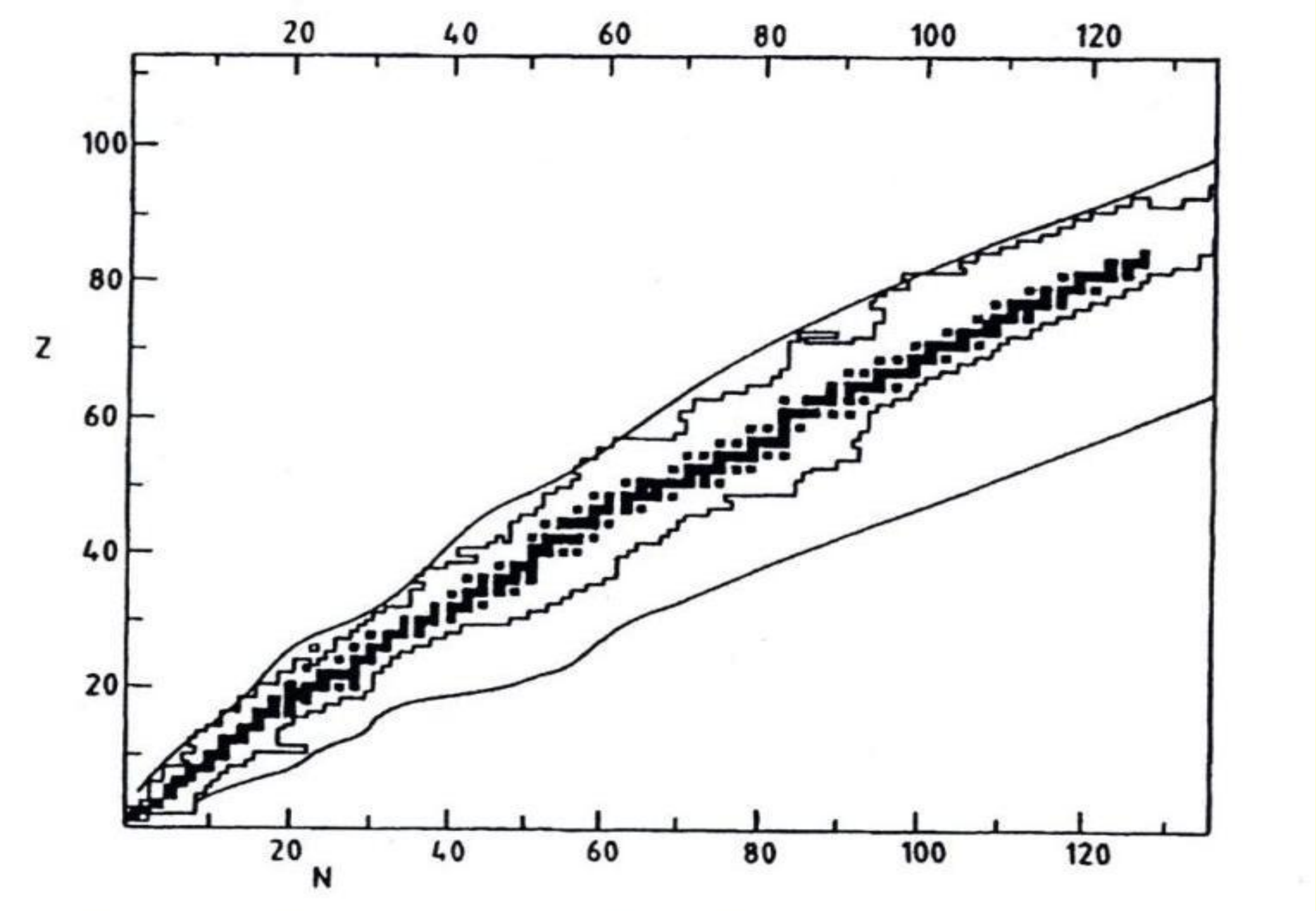
$$- U_0 + \alpha\delta(j)$$

Where,

δ = -*l* or *l* +1 if *j* = *l* +1/2 or *l*-1/2

Single-particle spectrum up to N=5. the various contributions to the full orbital and spin orbit splitting are presented. Partial and accumulated nucleon numbers are also given.





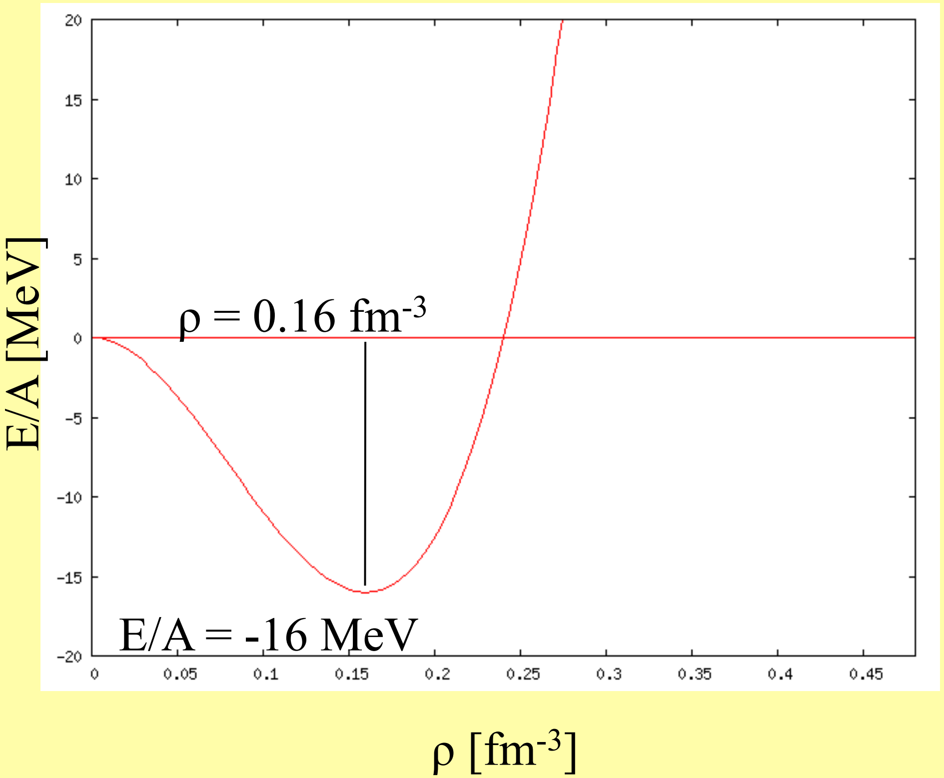
Map of the existing nuclei. The black squares in the central zone are stable nuclei, the broken inner lines show the status of known unstable nuclei as of 1986 and the outer lines are the assessed proton and neutron drip lines (Hansen 1991).

OBJECTIVE

1. Important task: Develop a modern Energy Density Functional (EDF), E = E[ρ], with enhanced predictive power for properties of rare nuclei.
2. We start from EDF obtained from the Skyrme N-N interaction.
3. The effective Skyrme interaction has been used in mean-field models for several decades. Many different parameterizations of the interaction have been realized to better reproduce nuclear masses, radii, and various other data. Today, there is more experimental data of nuclei far from the stability line. It is time to improve the parameters of Skyrme interactions. We fit our mean-field results to an extensive set of experimental data and obtain the parameters of the Skyrme type effective interaction for nuclei at and far from the stability line.

Equation of state and nuclear matter compressibility

The symmetric nuclear matter (N=Z and no Coulomb) incompressibility coefficient, *K,* is a important physical quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collisions, since it is directly related to the curvature of the nuclear matter (NM) equation of state (EOS), $E = E(\rho)$.



$$K = k_f^2 \left. \frac{d^2(E/A)}{dk_f^2} \right|_{k_{fo}} = 9\rho^2 \left. \frac{d^2(E/A)}{d\rho^2} \right|_{\rho_o}$$

$$E[\rho] = E[\rho_o] + \frac{1}{18} K \left(\frac{\rho - \rho_o}{\rho_o} \right)^2$$

$$E_{ANM}[\rho_o(\beta)] = E[\rho_o(\beta)] + \frac{1}{18} K(\rho_o(\beta)) \left(\frac{\rho - \rho_o(\beta)}{\rho_o(\beta)} \right)^2$$

$$E[\rho_o(\beta)] = E[\rho_o] + J\beta^2$$

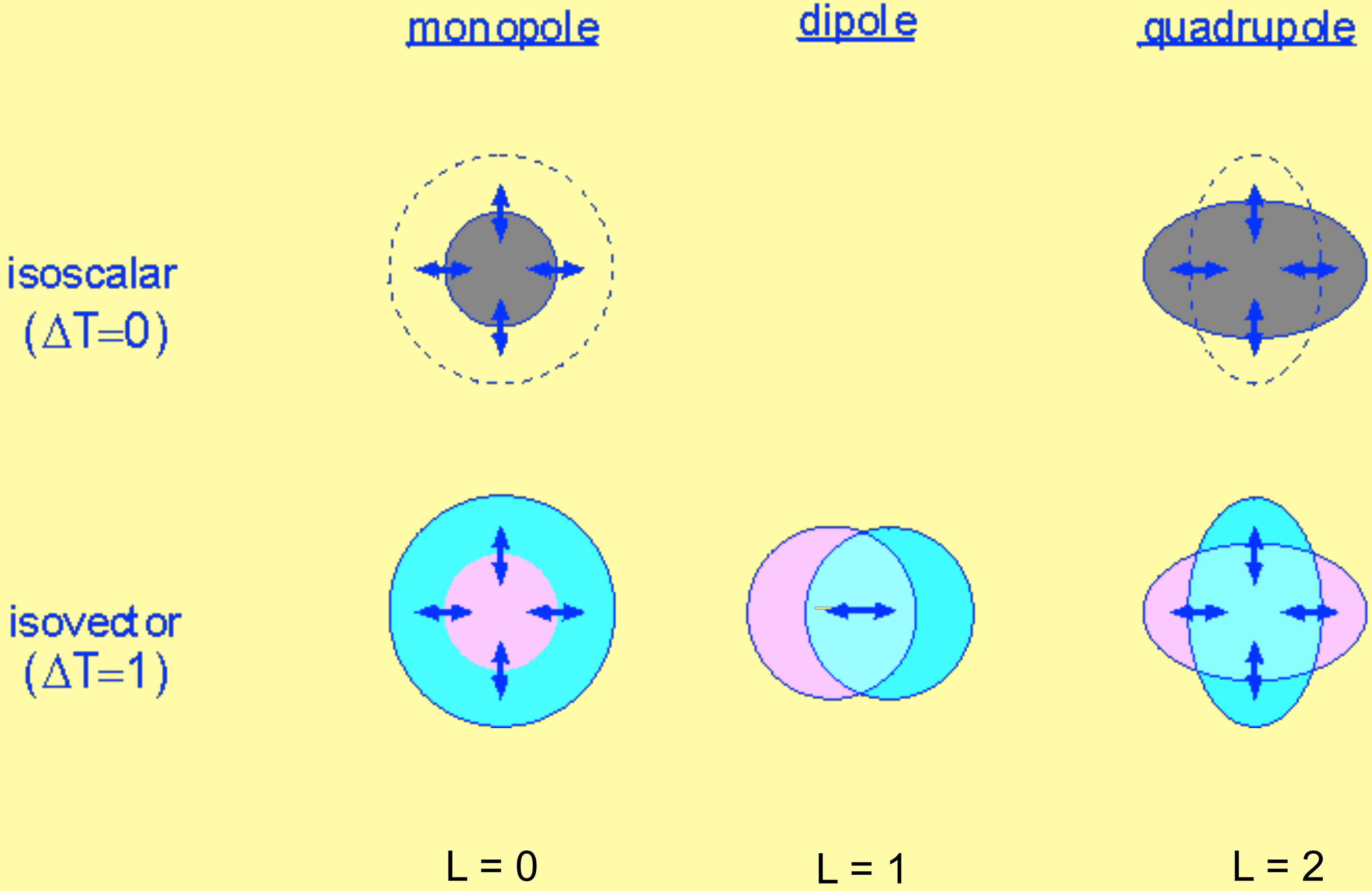
$$K[\rho_o(\beta)] = K + K_{\tau v}\beta^2$$

$$E_{SYM}(\rho) = \frac{1}{8} \left. \frac{d^2(E/A)}{dy^2} \right|_{\rho, y=1/2}$$

$$J = E_{SYM}[\rho_o]$$

$$\beta = (N - Z)/A \qquad y = Z/A$$

Macroscopic picture of giant resonance



Modern Energy Density Functional

Within the HF approximation: the ground state wave function Φ

$$\Phi = \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_1(\vec{r}_1,\sigma_1,\tau_1) & \phi_2(\vec{r}_1,\sigma_1,\tau_1) & \ldots & \phi_A(\vec{r}_1,\sigma_1,\tau_1) \\ \phi_1(\vec{r}_2,\sigma_2,\tau_2) & \phi_2(\vec{r}_2,\sigma_2,\tau_2) & \ldots & \phi_A(\vec{r}_2,\sigma_2,\tau_2) \\ \vdots & \vdots & \vdots & \vdots \\ \phi_1(\vec{r}_A,\sigma_A,\tau_A) & \phi_2(\vec{r}_A,\sigma_A,\tau_A) & \ldots & \phi_A(\vec{r}_A,\sigma_A,\tau_A) \end{vmatrix}$$

In spherical case

$$\phi_i(\vec{r},\sigma,\tau) = \frac{R_{\alpha_i}(r)}{r} Y_{jlm}(r,\sigma)\chi_{m_\tau}(\tau)$$

HF equations: minimize $E = \left\langle \Phi \middle| \hat{H}_{total} \middle| \Phi \right\rangle$

The total Hamiltonian of the nucleus

$$\hat{H}_{total} = T + V = \sum_{i=1}^{A} \frac{p_i^2}{2m_i} + \sum_{i\langle j=1} V\left(\vec{r}_i, \vec{r}_j\right)$$

where

$$V(\vec{r}_i, \vec{r}_j) = V_{ij}^{NN} + V_{ij}^{Coul}.$$

The total energy

$$E = \left\langle \Phi \middle| \hat{H}_{total} \middle| \Phi \right\rangle = -\frac{\hbar^2}{2m} \sum_{i=1}^{A} \int \phi_{\alpha_i}^*(\vec{r}) \Delta \phi_{\alpha_i}(\vec{r}) d\vec{r}$$

$$+ \sum_{i\langle j}^{A} \int \phi_{\alpha_i}^*(\vec{r}) \phi_{\alpha_j}^*(\vec{r}\,') V(\vec{r}, \vec{r}\,') \phi_{\alpha_i}(\vec{r}) \phi_{\alpha_j}(\vec{r}\,') d\vec{r} d\vec{r}\,'$$

$$- \sum_{i\langle j}^{A} \int \phi_{\alpha_i}^*(\vec{r}) \phi_{\alpha_j}^*(\vec{r}\,') V(\vec{r}, \vec{r}\,') \phi_{\alpha_i}(\vec{r}\,') \phi_{\alpha_j}(\vec{r}) d\vec{r} d\vec{r}\,'$$

Skyrme interaction

For the nucleon-nucleon interaction $V(\vec{r}_i,\vec{r}_j)=V_{ij}^{NN}+V_{ij}^{Coul}.$

$$V_{ij}^{Coul}=-\frac{e^2}{4}\sum_{i,j=1}^{A}\frac{\tau_{ij}^2+\tau_{ij}}{\left|\vec{r}_i-\vec{r}_j\right|},\qquad \tau_{ij}=\tau_i+\tau_j$$

V_{ij}^{NN} we adopt the standard Skyrme type interaction

$$V_{ij}^{NN}=t_0(1+x_0P_{ij}^{\sigma})\delta(\vec{r}_i-\vec{r}_j)+\frac{1}{2}t_1(1+x_1P_{ij}^{\sigma})[\overleftarrow{k}_{ij}^{2}\delta(\vec{r}_i-\vec{r}_j)+\delta(\vec{r}_i-\vec{r}_j)\vec{k}_{ij}^{2}]+$$

$$t_2(1+x_2P_{ij}^{\sigma})\overleftarrow{k}_{ij}\delta(\vec{r}_i-\vec{r}_j)\vec{k}_{ij}+\frac{1}{6}t_3(1+x_3P_{ij}^{\sigma})\rho^{\alpha}\left(\frac{\vec{r}_i+\vec{r}_j}{2}\right)\delta(\vec{r}_i-\vec{r}_j)+$$

$$iW_0\overleftarrow{k}_{ij}\delta(\vec{r}_i-\vec{r}_j)(\vec{\sigma}_i+\vec{\sigma}_j)\vec{k}_{ij},$$

$t_i,\ x_i,\ \alpha,\ W_0$ are 10 Skyrme parameters.

The total energy

$$E = \left\langle \Phi \middle| \hat{H}_{total} \middle| \Phi \right\rangle = \left\langle \Phi \middle| T + V_{Coulomb} + V_{12} \middle| \Phi \right\rangle = \int H(\vec{r})d\vec{r}$$

where

$$H(\vec{r}) = H_{Kinetic}(\vec{r}) + H_{Coulomb}(\vec{r}) + H_{Skyrme}(\vec{r})$$

$$H_{Kinetic}(\vec{r}) = \frac{\hbar^2}{2m_p}\tau_p(\vec{r}) + \frac{\hbar^2}{2m_n}\tau_n(\vec{r})$$

$$H_{Coulomb}(\vec{r}) = \frac{e^2}{2}\left[\rho_{ch}(\vec{r})\int \frac{\rho_{ch}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}d\vec{r}\,' - \int \frac{|\rho_{ch}(\vec{r},\vec{r}\,')|^2}{|\vec{r}-\vec{r}\,'|}d\vec{r}\,'\right]$$

$$H_{Skyrme}(\vec{r}) = H_0 + H_3 + H_{eff} + H_{fin} + H_{so} + H_{sg}$$

$$\mathcal{H}_0 = \frac{1}{4}t_0\left[(2+x_0)\rho^2 - (2x_0+1)(\rho_p^2+\rho_n^2)\right],$$

$$\mathcal{H}_3 = \frac{1}{24}t_3\rho^\alpha\left[(2+x_3)\rho^2 - (2x_3+1)(\rho_p^2+\rho_n^2)\right],$$

$$\mathcal{H}_{\text{eff}} = \frac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\tau\rho + \frac{1}{8}\left[t_2(2x_2+1)-t_1(2x_1+1)\right](\tau_p\rho_p+\tau_n\rho_n),$$

$$\begin{aligned}
\mathcal{H}_{\text{fin}} &= \frac{1}{32}\left[3t_1(2+x_1)-t_2(2+x_2)\right](\nabla\rho)^2 \\
&\quad -\frac{1}{32}\left[3t_1(2x_1+1)+t_2(2x_2+1)\right]\left[(\overrightarrow{\nabla}\rho_p)^2+(\overrightarrow{\nabla}\rho_n)^2\right], \\
\mathcal{H}_{\text{so}} &= \frac{W_0}{2}\left[\mathbf{J}\cdot\nabla\rho+\mathbf{J}_p\cdot\nabla\rho_p+\mathbf{J}_n\cdot\nabla\rho_n\right], \\
\mathcal{H}_{\text{sg}} &= -\frac{1}{16}(t_1x_1+t_2x_2)\mathbf{J}^2+\frac{1}{16}(t_1-t_2)\left[\mathbf{J}_p^2+\mathbf{J}_n^2\right].
\end{aligned}$$

$$\rho(\vec{r}) = \sum_{\tau} \rho_{\tau}(\vec{r}) \qquad \rho_{\tau}(\vec{r}) = \sum_{i=1}^{A} \sum_{\sigma} \phi_i^{*}(\vec{r},\sigma,\tau)\phi_i(\vec{r},\sigma,\tau)$$

$$\tau(\vec{r}) = \sum_{\tau} \tau_{\tau}(\vec{r}) \qquad \tau_{\tau}(\vec{r}) = \sum_{i=1}^{A} \sum_{\sigma} \left|\vec{\nabla}\phi_i(\vec{r},\sigma,\tau)\right|^2$$

$$\vec{J}(\vec{r}) = \sum_{\tau} \vec{J}_{\tau}(\vec{r}) \qquad \vec{J}_{\tau}(\vec{r}) = -i\sum_{i=1}^{A} \sum_{\sigma,\sigma'} \phi_i^{*}(\vec{r},\sigma,\tau)\left[\vec{\nabla}\phi_i(\vec{r},\sigma',\tau)\times\langle\sigma|\vec{\sigma}|\sigma'\rangle\right]$$

$$\rho_{ch}(\vec{r},\vec{r}') = \sum_{i,\sigma,\sigma'} \phi_i^{*}(\vec{r},\sigma,\frac{1}{2})\phi_i(\vec{r}',\sigma',\frac{1}{2})$$

Now we apply the variation principle to derive the Hartree-Fock equations. We minimize the Energy E, given in terms of the energy density functional

$$E = \left\langle \Phi \middle| \hat{H}_{total} \middle| \Phi \right\rangle = \int H(\vec{r})\, d\vec{r}$$

$$\frac{\delta}{\delta\rho_{\sigma,\tau}}\left[E - \sum_i \varepsilon_i \int \rho_{\sigma,\tau} d\vec{r}\right] = \frac{\delta E}{\delta\rho_{\sigma,\tau}} - \frac{\delta\left[\sum_i \varepsilon_i \int \rho_{\sigma,\tau} d\vec{r}\right]}{\delta\rho_{\sigma,\tau}} = 0 \qquad (*)$$

where

$$\delta E = \sum_{\sigma,\tau}\int\left[\frac{\hbar^2}{2m_\tau^*(\vec{r})}\delta\tau_{\sigma\tau}(\vec{r}) + U_\tau(\vec{r})\delta\rho_{\sigma\tau}(\vec{r}) + \vec{W}_\tau(\vec{r})\delta\vec{J}_{\sigma\tau}(\vec{r})\right]d\vec{r}$$

$$\delta\rho_{\sigma\tau} = \sum_{i,\sigma'}\phi_i(\vec{r},\sigma',\tau)\delta\phi_i^*(\vec{r},\sigma',\tau)$$

$$\delta\tau_{\sigma\tau}(\vec{r}) = \sum_{i,\sigma'}\vec{\nabla}\phi_i(\vec{r},\sigma',\tau)\vec{\nabla}\delta\phi_i^*(\vec{r},\sigma',\tau)$$

$$\delta\vec{J}_{\sigma\tau}(\vec{r}) = -i\sum_{i,\sigma',\sigma''}\delta\phi_i^*(\vec{r},\sigma',\tau)\left[\vec{\nabla}\phi_i(r,\sigma'',\tau)\times\langle\sigma'|\vec{\sigma}|\sigma''\rangle\right]$$

After carrying out the minimization of energy, we obtain the HF equations:

$$\frac{\hbar^2}{2m_\tau^*(r)}\left[-R_\alpha''(r)+\frac{l_\alpha(l_\alpha+1)}{r^2}R_\alpha(r)\right]-\frac{d}{dr}\left(\frac{\hbar^2}{2m_\tau^*(r)}\right)R_\alpha'(r)$$

$$+\left[U_\tau(r)+\frac{1}{r}\frac{d}{dr}\left(\frac{\hbar^2}{2m_\tau^*(r)}\right)+\frac{\left[j_\alpha(j_\alpha+1)-l_\alpha(l_\alpha+1)-\frac{3}{4}\right]}{r}W_\tau(r)\right]R_\alpha(r)$$

$$=\varepsilon_\alpha R_\alpha(r)$$

where $m_\tau^*(r)$, $U_\tau(r)$, and $W_\tau(r)$ are the effective mass, the potential and the spin orbit potential. They are given in terms of the Skyrme parameters and the nuclear densities.

$$\frac{\hbar^2}{2m_\tau^*(\vec{r})} = \frac{\hbar^2}{2m_\tau} + \frac{1}{4}\left[t_1(1+\frac{1}{2}x_1) + t_2(1+\frac{1}{2}x_2)\right]\rho(\vec{r}) - \frac{1}{4}\left[t_1(\frac{1}{2}+x_1) - t_2(\frac{1}{2}+x_2)\right]\rho_\tau(\vec{r})$$

$$U_\tau(\vec{r}) = t_0(1+\frac{1}{2}x_0)\rho(\vec{r}) - t_0(\frac{1}{2}+x_0)\rho_\tau(\vec{r}) + \frac{1}{4}\left[t_1(1+\frac{1}{2}x_1) + t_2(1+\frac{1}{2}x_2)\right]\tau(\vec{r})$$
$$-\frac{1}{4}\left[t_1(\frac{1}{2}+x_1) - t_2(\frac{1}{2}+x_2)\right]\tau_\tau(\vec{r}) + \frac{\alpha+2}{12}t_3(1+\frac{1}{2}x_3)\rho^{\alpha+1}(\vec{r})$$
$$-\frac{\sigma}{12}t_3(\frac{1}{2}+x_3)\rho^{\alpha-1}(\vec{r})\left(\rho_\tau^2(\vec{r}) + \rho_{-\tau}^2(\vec{r})\right) - \frac{1}{6}t_3(\frac{1}{2}+x_3)\rho^{\alpha}(\vec{r})\rho_\tau$$
$$-\frac{1}{8}\left[3t_1(1+\frac{1}{2}x_1) - t_2(1+\frac{1}{2}x_2)\right]\nabla^2\rho(\vec{r}) + \frac{1}{8}\left[3t_1(\frac{1}{2}+x_1) + t_2(\frac{1}{2}+x_2)\right]\nabla^2\rho_\tau(\vec{r})$$
$$-\frac{1}{2}W_0\left[\nabla\vec{J}(\vec{r}) + \nabla\vec{J}_\tau(\vec{r})\right] + \delta_{\tau,\frac{1}{2}}\, e^2\int d\vec{r}\,'\frac{\rho_{ch.}(\vec{r}\,')}{\left|\vec{r}-\vec{r}\,'\right|},$$

$$W_\tau(\vec{r}) = \frac{1}{2}W_0\left[\nabla\vec{\rho}(\vec{r}) + \nabla\vec{\rho}_\tau(\vec{r})\right] + \frac{1}{8}(t_1 - t_2)\vec{J}_\tau(\vec{r}) - \frac{1}{8}[t_1x_1 - t_2x_2]\vec{J}(\vec{r})$$

With an initial guess of the single-particle wave functions (example; harmonic oscillator wave functions), we can determine m^*, $U(r)$, and $W(r)$ and solve the HF equation to get a set of new single-particle wave functions; then one can proceed in this way until reaching convergence.

NOTES:

1. One should start close to the solution.
2. Accuracy and convergence in three dimension

3 Convergence of HFB equations in three dimension?

Infinite Nuclear Matter

It's important to note that the proton and neutron densities are constants.

So that $$\frac{E(\rho_n,\rho_p)}{A} = \frac{1}{A}\int H(\boldsymbol{r})d\boldsymbol{r} = \frac{\varepsilon(\rho_n,\rho_p)}{A}V = \frac{\varepsilon(\rho_n,\rho_p)}{\rho}$$

where ε is the energy density functional

The EOS of asymmetric NM, with $Y_p = Z/A$ (or $I = (N - Z)/A$)

$$\frac{E}{A}(Y_p \text{ or } I, \rho) = \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{2/3}F_{5/3} + \frac{1}{8}t_0\rho\left[2(x_0+2) - (2x_0+1)F_2\right]$$
$$+\frac{1}{48}t_3\rho^{\sigma+1}\left[2(x_3+2) - (2x_3+1)F_2\right]$$
$$+\frac{3}{40}\left(\frac{3\pi^2}{2}\right)^{2/3}\rho^{5/3}\left\{\left[t_1(x_1+2) + t_2(x_2+2)\right]F_{5/3}\right.$$
$$\left.+\frac{1}{2}\left[t_2(2x_2+1) - t_1(2x_1+1)\right]F_{8/3}\right\},$$

Note that $\sigma = \alpha$

with $F_m(Y_p) = 2^{m-1}\left[Y_p^m + (1-Y_p)^m\right]$, $\quad F_m(I) = \frac{1}{2}\left[(1+I)^m + (1-I)^m\right]$

Determining the Skyrme interaction using the HF approach

Approximations

Coulomb energy

$$H_{Coulomb} = \frac{1}{2} V_{Coul}^{dir}(r)\rho_p(r) + \frac{3}{4} V_{Coul}^{ex}(r)\rho_p(r)$$

$$V_{Coul}^{dir} = e^2 \int \frac{\rho_p(r')d^3r'}{|\vec{r} - \vec{r}'|} \qquad V_{Coul}^{ex} = -e^2 \left(\frac{3\rho_p(r)}{\pi} \right)^{1/3}$$

Note that here the direct term and exchange Coulomb terms each include the spurious self-interaction term.

Interaction: KDE0, KDE0v1 neglect exchange term

KDE, include exchange term

KDEX, include contributions of g.s. correlations

Center of mass correction

a). Correction to the total binding energy:

We use the harmonic oscillator approximation. The CM energy is taken as

$K_{CM}^{osc} = \frac{3}{4}\hbar\omega$ but $\hbar\omega$ Is determined by using the mass mean-square radii $\langle r^2 \rangle$

$$\hbar\omega = \frac{\hbar^2}{mA\langle r^2 \rangle}\sum_i \left[N_i + \frac{3}{2}\right]$$

b). Correction to the charge rms radii r_{ch}

The charge mean-square radius to be fitted to the experimental data is obtained as

$$\langle r_{ch}^2 \rangle = \langle r_p^2 \rangle_{HF} - \frac{3}{2\upsilon A} + \langle r^2 \rangle_p + \frac{N}{Z}\langle r^2 \rangle_n + \frac{1}{Z}\left(\frac{\hbar}{mc}\right)\sum nlj\tau(2j+1)\mu_\tau \langle \sigma.l \rangle_{lj}$$

Simulated Annealing Method (SAM)

The SAM is a method for optimization problems of large scale, in particular, where a desired global extremum is hidden among many local extrema.

We use the SAM to determine the values of the Skyrme parameters by searching the global minimum for the chi-square function

$$\chi^2 = \frac{1}{N_d - N_p} \sum_{i=1}^{N_d} \left(\frac{M_i^{\exp} - M_i^{th}}{\sigma_i} \right)^2$$

N_d is the number of experimental data points.

N_p is the number of parameters to be fitted.

$M_i^{\exp}$ and M_i^{th} are the experimental and the corresponding theoretical values of the physical quantities.

σ_i is the adopted uncertainty.

Implementing the SAM to search the global minimum of χ^2 function:

1. $t_i,\ x_i,\ \alpha,\ W_0$ are written in term of $B/A,\ \ K_{nm},\ \ \rho_{nm},...$

2. Define $\vec{v}(B/A, K_{nm}, \rho_{nm}, m^*/m, E_s, J, L, \kappa, G_0^{'}, W_0)$

3. Calculate χ^2_{old} for a given set of experimental data and the corresponding HF results (using an initial guess for Skyrme parameters).

.

4. Determine a new set of Skyrme parameters by the following steps:

   + Use a random number to select a component v_r of vector $\vec{v}$

   + Use another random number η to get a new value of v_r

$$v_r \rightarrow v_r + d\eta$$

   + Use this modified vector $\vec{v}$ to generate a new set of Skyrme parameters.

5. Go back to HF and calculate χ^2_{new}

6. The new set of Skyrme parameters is accepted only if

$$P(\chi^2) = \exp\left(\frac{\chi^2_{old} - \chi^2_{new}}{T}\right) > \beta$$

$$0 < \beta < 1$$

7. Starting with an initial value of $T = T_i$, we repeat steps 4 - 6 for a large number of loops.

8. Reduce the parameter T as $T = \frac{T_i}{k}$ and repeat steps 1 – 7.

9. Repeat this until hopefully reaching global minimum of χ^2

Fitted data

- The binding energies for 14 nuclei ranging from normal to the exotic (proton or neutron) ones: ^{16}O, **^{24}O**, **^{34}Si**, ^{40}Ca, ^{48}Ca, **^{48}N**i, ^{56}Ni, ^{68}Ni, ^{78}Ni, ^{88}Sr, ^{90}Zr, **^{100}Sn**, ^{132}Sn, and ^{208}Pb.

- Charge rms radii for 7 nuclei: ^{16}O, ^{40}Ca, ^{48}Ca, ^{56}Ni, **^{88}Sr**, ^{90}Zr, ^{208}Pb.

- **The spin-orbit splittings for 2*p* proton and neutron orbits for ^{56}Ni $\varepsilon(2p_{1/2}) - \varepsilon(2p_{3/2})$ = 1.88 MeV (neutron)**
$\varepsilon(2p_{1/2}) - \varepsilon(2p_{3/2})$ = 1.83 MeV (proton).

- **Rms radii for the valence neutron:**

in the $1d_{5/2}$ orbit for ^{17}O $r_n(1d_{5/2}) = 3.36\,fm$

in the $1f_{7/2}$ orbit for ^{41}Ca $r_n(1f_{7/2}) = 3.99\,fm$

- **The breathing mode energy for 4 nuclei: ^{90}Zr (17.81 MeV), ^{116}Sn (15.9 MeV), ^{144}Sm (15.25 MeV), and ^{208}Pb (14.18 MeV).**

Constraints

1. The critical density $2\rho_0 < \rho_{cr} < 3\rho_0$

$$V_{p-h}^{Landau} = \sum_l \left(F_l + F_l^{'}\tau_1\tau_2 + G_l\sigma_1\sigma_2 + G_l^{'}\sigma_1\sigma_2\tau_1\tau_2\right)\delta\left(\vec{r}_1 - \vec{r}_2\right)$$

Landau stability condition: $F_l, F_l^{'}, G_l, G_l^{'} > -(2l+1)$

Example: $K = 6\frac{\hbar^2 k_F^2}{2m}\left(1+F_0\right)\Big/\left(1+F_1/3\right)$

2. The Landau parameter $G_0^{'}$ should be positive at $\rho = \rho_0$

3. The quantity $P = 3\rho\frac{dS}{d\rho}$ must be positive for densities up to $3\rho_0$

4. The IVGDR enhancement factor $0.25 < \kappa < 0.5$

$$\int ES_{L=1}^{T=1}(E)dE = \frac{\hbar^2}{2m}\frac{NZ}{A}(1+\kappa)$$

Self-consistent calculation within constrained HF

$H_\lambda = H - \lambda f$ monopole $f = \sum_{i=1}^{A} r_i^2$

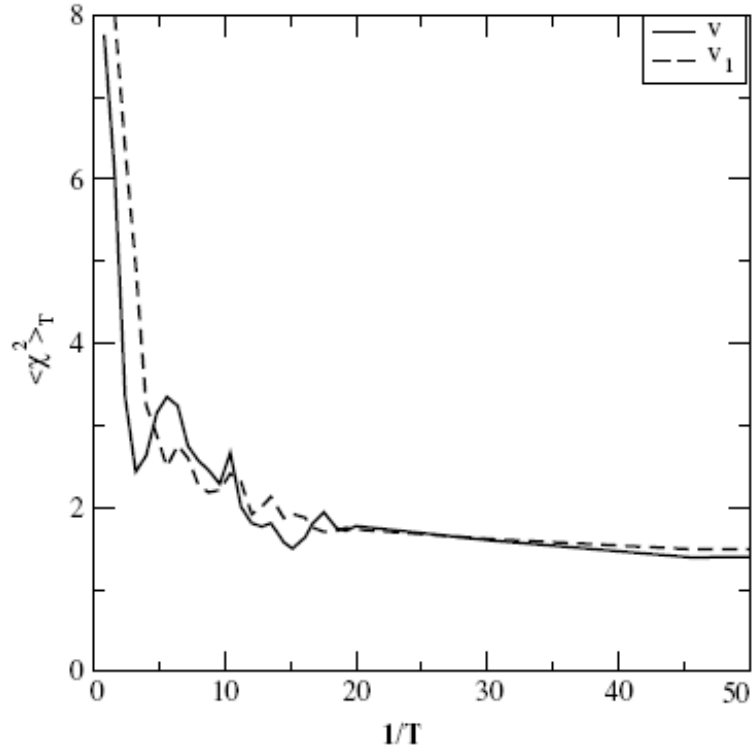
$E_{constr.} = (m_1/m_{-1})^{1/2} \quad E_s = (m_3/m_1)^{1/2}$

$m_k = \int E^k S(E) dE \quad m_1 = 2\frac{\hbar}{m}\langle r^2 \rangle$

$m_{-1} = \frac{1}{2}\frac{d}{d\lambda}\langle r_\lambda^2 \rangle\Big|_{\lambda=0} = \frac{1}{2}\frac{d^2}{d\lambda^2}\langle H_\lambda \rangle\Big|_{\lambda=0}$

$m_3 = \frac{1}{2}(\frac{2\hbar}{m})^2[2T + E_\delta + 20(E_{fin} + E_{s.o.}) + (3\alpha + 2)(3\alpha + 3)E_\rho]$

	v	v_0	v_1	d
B/A (MeV)	16.0	17.0	15.0	0.4
K_{nm} (MeV)	230.0	200.0	300.0	20.0
ρ_{nm} (fm^{-3})	0.160	0.150	0.170	0.005
m*/m	0.70	0.60	0.90	0.04
E_s (MeV)	18.0	17.0	19.0	0.3
J (MeV)	32.0	25.0	40.0	4.0
L (MeV)	47.0	20.0	80.0	10.0
Kappa	0.25	0.1	0.5	0.1
G'_0	0.08	0.00	0.40	0.10
W_0 (MeV fm^5)	120.0	100.0	150.0	5.0



Variation of the average value of $\langle \chi^2 \rangle_T$ as a function of the inverse of the control parameter T for the KDE0 interaction for the two different choices of the starting parameter.

Values of the Skyrme parameters and the corresponding physical quantities of nuclear matter for the KDE0 and KDE0v1 and KDEX interactions.

Parameter	KDE0	KDE0v1	KDEX
t_0 (MeV fm^3)	-2526.5110	-2553.0843	-1419.8304
t_1 (MeV fm^5)	430.9418	411.6963	309.1373
t_2 (MeV fm^5)	-398.3775	-419.8712	-172.9562
t_3(MeVfm$^{3(1+\alpha)}$)	14235.5193	14603.6069	10465.3523
x_0	0.7583	0.6483	0.1474
x_1	-0.3087	-0.3472	-0.0853
x_2	-0.9495	-0.9268	-0.6144
x_3	1.1445	0.9475	0.0220
W_0(MeV fm^5)	128.9649	124.4100	98.8973
α	0.1676	0.1673	0.4989
B/A (MeV)	16.11	16.23	15.96
K (MeV)	228.82	227.54	274.20
ρ_0 (fm^{-3})	0.161	0.165	0.155
m*/m	0.72	0.74	0.81
J (MeV)	33.00	34.58	32.76
L (MeV)	45.22	54.69	63.70
κ	0.30	0.23	0.33
G'_0	0.05	0.00	0.41

Binding Energies (MeV)

		$\Delta B = B^{exp} - B^{th}$	
Nuclei	B^{exp}	KDE0	KDEX
^{16}O	-127.620	0.394	3.202
^{24}O	-168.384	-0.581	4.582
^{34}Si	-283.427	-0.656	2.868
^{40}Ca	-342.050	0.005	0.699
^{48}Ca	-415.990	0.188	2.529
^{48}Ni	-347.136	-1.437	4.946
^{56}Ni	-483.991	1.091	1.853
^{68}Ni	-590.408	0.169	1.532
^{78}Ni	-641.940	-0.252	2.597
^{88}Sr	-768.468	0.826	2.985
^{90}Zr	-783.892	-0.127	0.913
^{100}Sn	-824.800	-3.664	0.180
^{132}Sn	-1102.850	-0.422	1.752
^{208}Pb	-1636.430	0.945	-5.584

G. Audi et al, Nucl. Phys. **A729**, 337 (2003)

Charge RMS Radii (fm)

E. W. Otten, in Treatise onn Heavy-Ion Science, Vol 8 (1989).

H. D. Vries et al, At. Data Nucl. Tables **36**, 495 (1987).

F. Le Blanc et al, Phys. Rev. C **72**, 034305 (2005).

Nuclei	Experiment	KDE0	KDEX
^{16}O	2.73	2.771	2.713
^{40}Ca	3.49	3.490	3.456
^{48}Ca	3.48	3.501	3.485
^{56}Ni	3.75	3.768	3.848
^{88}Sr	4.219	4.221	4.213
^{90}Zr	4.258	4.266	4.261
^{132}Sn	4.709	4.710	4.717
^{208}Pb	5.500	5.489	5.499

Single-Particle Energies (in MeV) for ^{40}Ca

Orbits	Expt.	KDE0*	Orbits	Expt.	KDE0*
Protons			Neutrons		
$1s_{1/2}$	-50 ± 11	-38.21	$1s_{1/2}$	-	-47.77
$1p_{3/2}$	-	-26.42	$1p_{3/2}$	-	-34.90
$1p_{1/2}$	-34 ± 6	-22.34	$1p_{1/2}$	-	-30.78
$1d_{5/2}$	-	-14.51	$1d_{5/2}$	-	-22.08
$2s_{1/2}$	-10.9	-9.66	$2s_{1/2}$	-18.1	-17.00
$1d_{3/2}$	-8.3	-7.53	$1d_{3/2}$	-15.6	-14.97
$1f_{7/2}$	-1.4	-2.76	$1f_{7/2}$	-8.3	-9.60
			$2p_{3/2}$	-6.2	-4.98

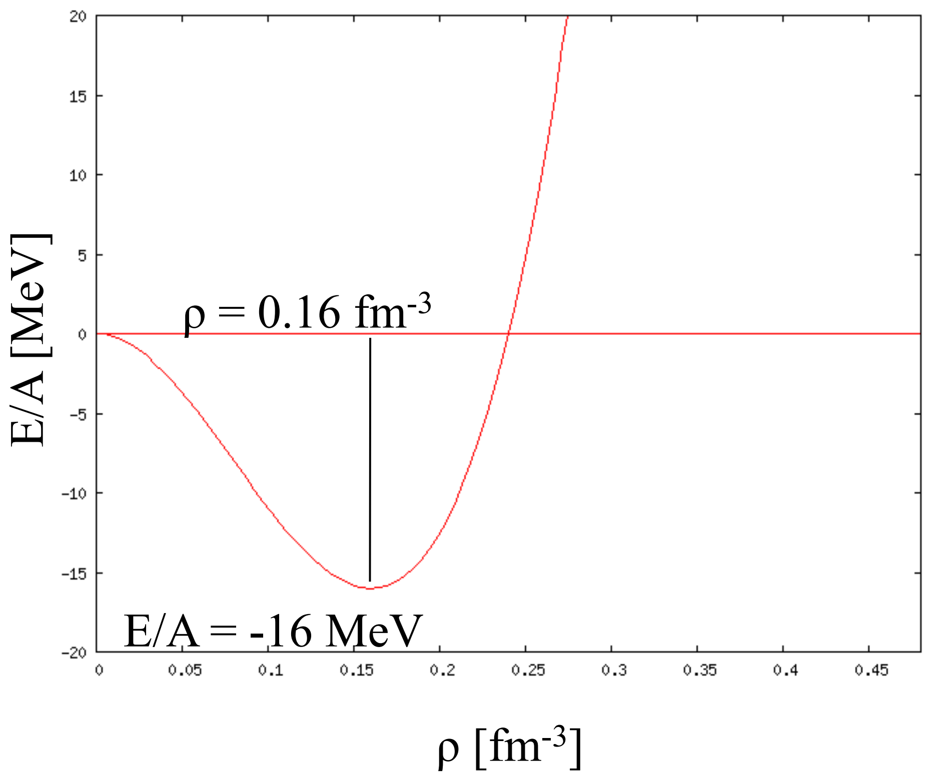
*TAMU Skyrme Interaction: B. K. Agrawal, S. Shlomo and V. Kim Au, Phys. Rev. C **72**, 014310 (2005).

GIANT RESONANCES

- Hadron Scattering
- HF-Based RPA
- Results

Equation of state and nuclear matter compressibility

The symmetric nuclear matter (N=Z and no Coulomb) incompressibility coefficient, *K*, is a important physical quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collisions, since it is directly related to the curvature of the nuclear matter (NM) equation of state (EOS), $E = E(\rho)$.



$$K = k_f^2 \frac{d^2(E/A)}{dk_f^2}\bigg|_{k_{fo}} = 9\rho^2 \frac{d^2(E/A)}{d\rho^2}\bigg|_{\rho_o}$$

$$E[\rho] = E[\rho_o] + \frac{1}{18} K \left(\frac{\rho - \rho_o}{\rho_o}\right)^2$$

$$E_{ANM}[\rho_o(\beta)] = E[\rho_o(\beta)] + \frac{1}{18} K(\rho_o(\beta)) \left(\frac{\rho - \rho_o(\beta)}{\rho_o(\beta)}\right)^2$$

$$E[\rho_o(\beta)] = E[\rho_o] + J\beta^2$$

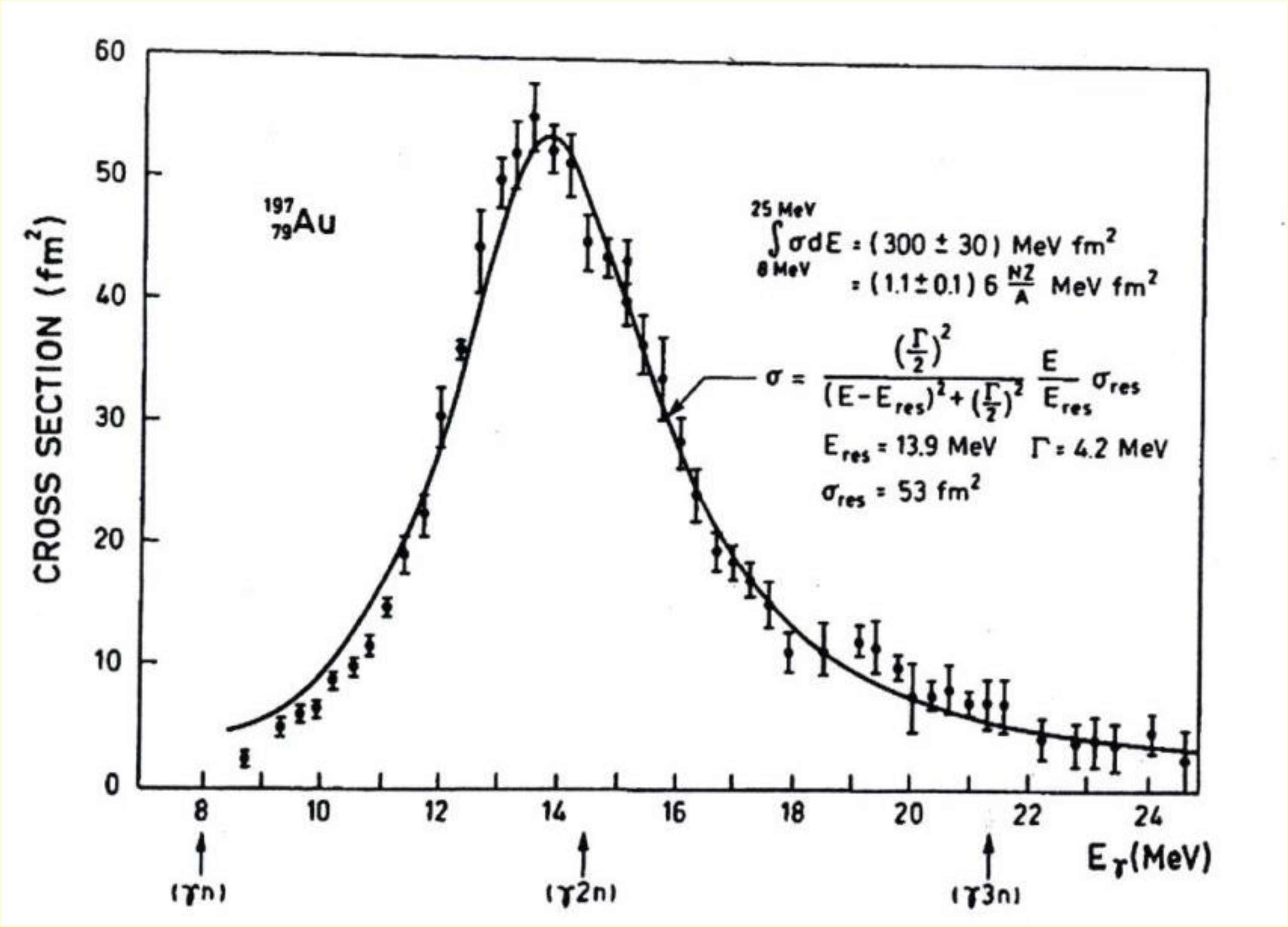
$$K[\rho_o(\beta)] = K + K_{\tau v}\beta^2$$

$$E_{SYM}(\rho) = \frac{1}{8} \frac{d^2(E/A)}{dy^2}\bigg|_{\rho, y=1/2}$$

$$J = E_{SYM}[\rho_o]$$

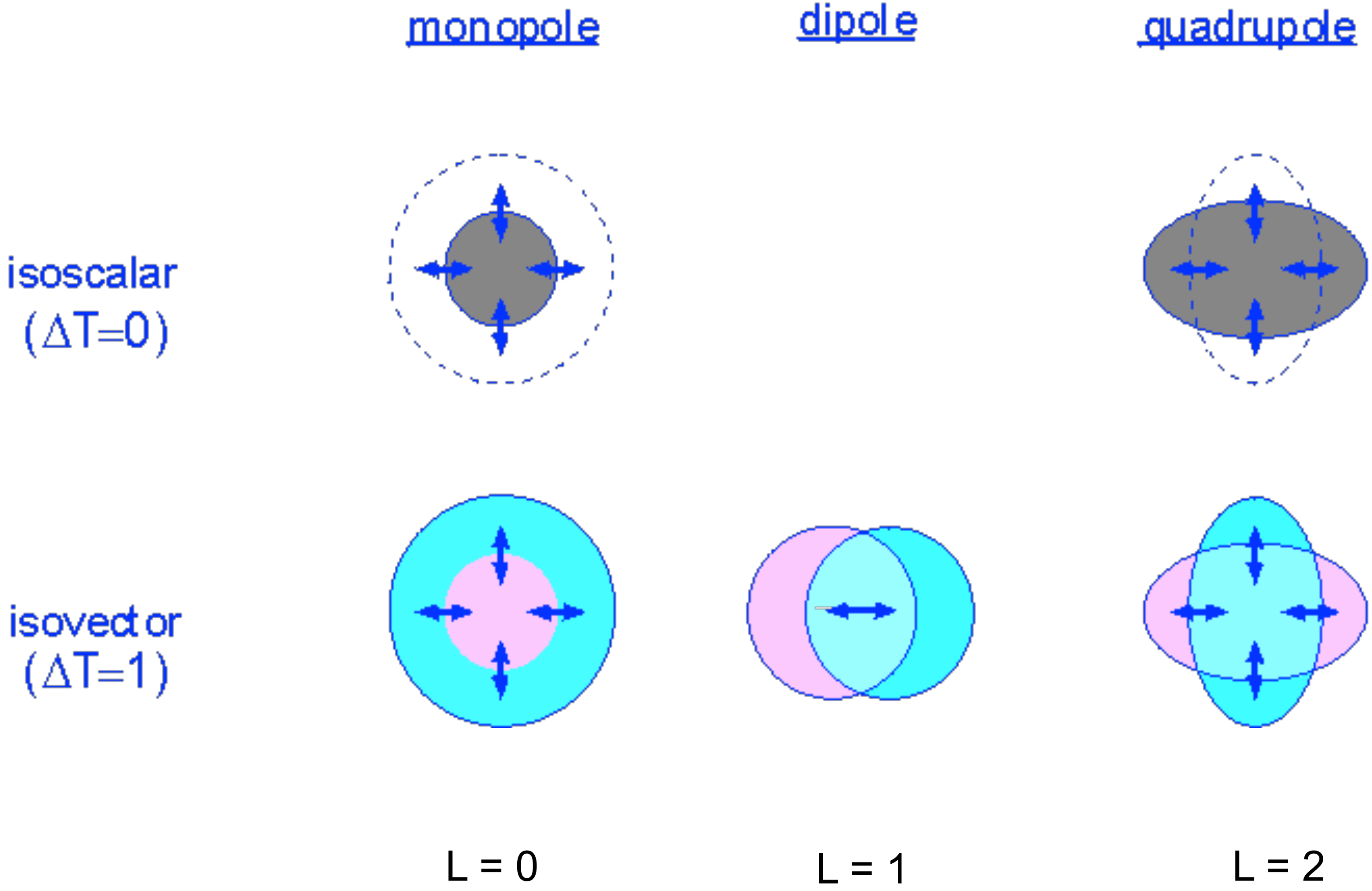
$$\beta = (N - Z)/A \qquad y = Z/A$$

The isovector giant dipole resonance

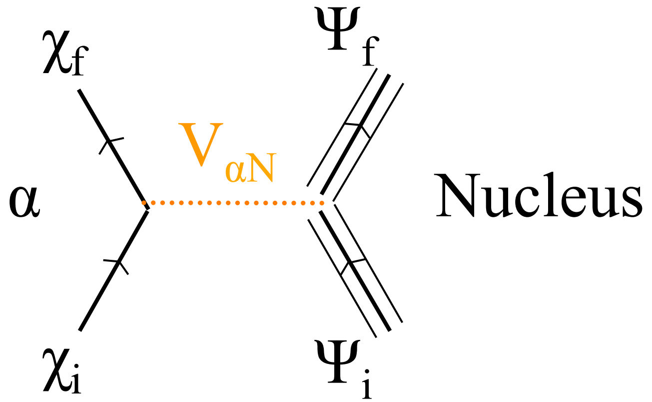


The total photoabsorption cross-section for ^{197}Au, illustrating the absorption of photons on a giant resonating electric dipole state. The solid curve show a Breit-Wigner shape. (Bohr and Mottelson, *Nuclear Structure*, vol. 2, 1975).

Macroscopic picture of giant resonance



Hadron excitation of giant resonances



$$\frac{d\sigma^{DWBA}}{d\Omega} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i}|T_{fi}|^2$$

$$T_{fi} = \langle \chi_f^{(-)} \Psi_f | V | \chi_i^{(+)} \Psi_i \rangle = \begin{cases} \langle \Psi_f | \mathcal{O}_{sc} | \Psi_i \rangle, & \mathcal{O}_{sc} \sim \int \chi_f^{(-)*} V \chi_i^{(+)} \\ \langle \chi_f^{(-)} | U_{tr} | \chi_i^{(+)} \rangle, & U_{tr} \sim \int \Psi_f^* V \Psi_i. \end{cases}$$

Theorists: calculate transition strength S(E) within HF-RPA using a simple scattering operator $F \sim r^L Y_{LM}$:

$$S(E) = \sum_n |\langle \Psi_0 | F | \Psi_n \rangle|^2 \delta(E - E_n)$$

Experimentalists: calculate cross sections within Distorted Wave Born Approximation (DWBA):

$$\frac{d\sigma(E)}{d\Omega} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \frac{k_f}{k_i} |\langle \chi_f^{(-)} | U_{tr} | \chi_i^{(+)} \rangle|^2$$

$$U_{tr} \sim \frac{\partial U}{\partial r}$$

or using folding model.

DWBA-Folding model description

1. Folding model optical potential:

$$U(\vec{r}) = \int d\vec{r}' \; V(|\vec{r}-\vec{r}'|, \rho_0(r'))\rho_0(r')$$

$$V(|\vec{r}-\vec{r}'|, \rho_0(r')) = -V(1+\beta_V \rho_0^{2/3}(r'))exp(-\frac{|\vec{r}-\vec{r}'|^2}{\alpha_V})$$
$$-iW(1+\beta_W \rho_0^{2/3}(r'))exp(-\frac{|\vec{r}-\vec{r}'|^2}{\alpha_W}).$$

Parameters of V determined by fit to elastic scattering cross section.

2. Transition potential for the DWBA calculation of excitation cross section of state with multipolarity L:

$$\delta U(r,E) = \int d\vec{r}' \delta\rho_L(\vec{r}', E)[\quad V(|\vec{r}-\vec{r}'|, \rho_0(r')) + \rho_0(r')\frac{\partial V(|\vec{r}-\vec{r}'|, \rho_0(r'))}{\partial \rho_0(r')}].$$

3. Transition density:

- Microscopic approach: from HF-RPA
- Collective model approach:

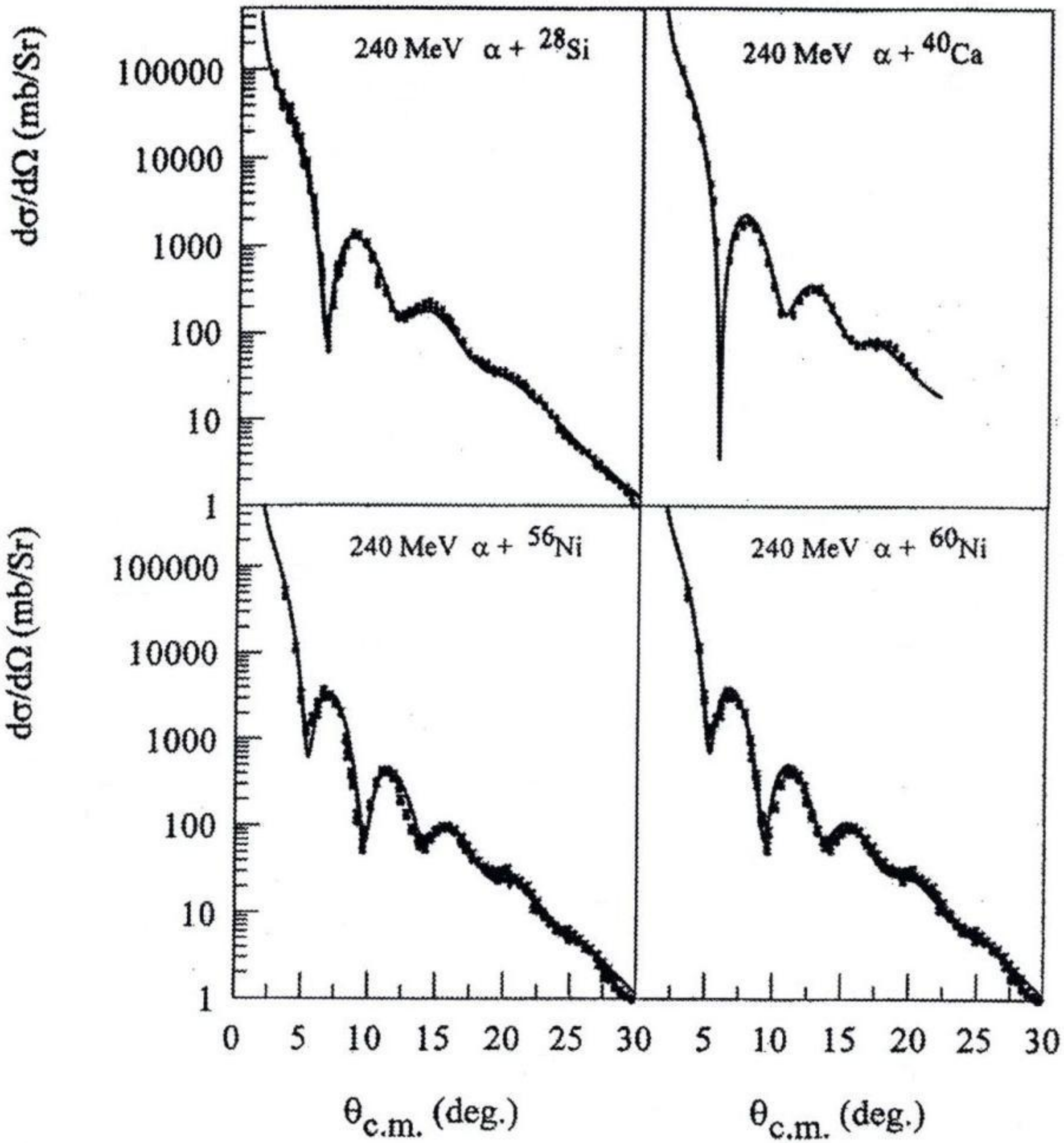
$$\delta\rho_{L=0}(r) = -\alpha(E)(3\rho_0(r) + r\frac{d\rho_0(r)}{r}),$$

$$\delta\rho_{L=1}(r) = -\alpha_{L=1}(E)(3r^2\frac{d\rho_0(r)}{dr} + 10r\rho_0(r) - \frac{5}{3}\langle r^2\rangle\frac{d\rho_0(r)}{dr}),$$

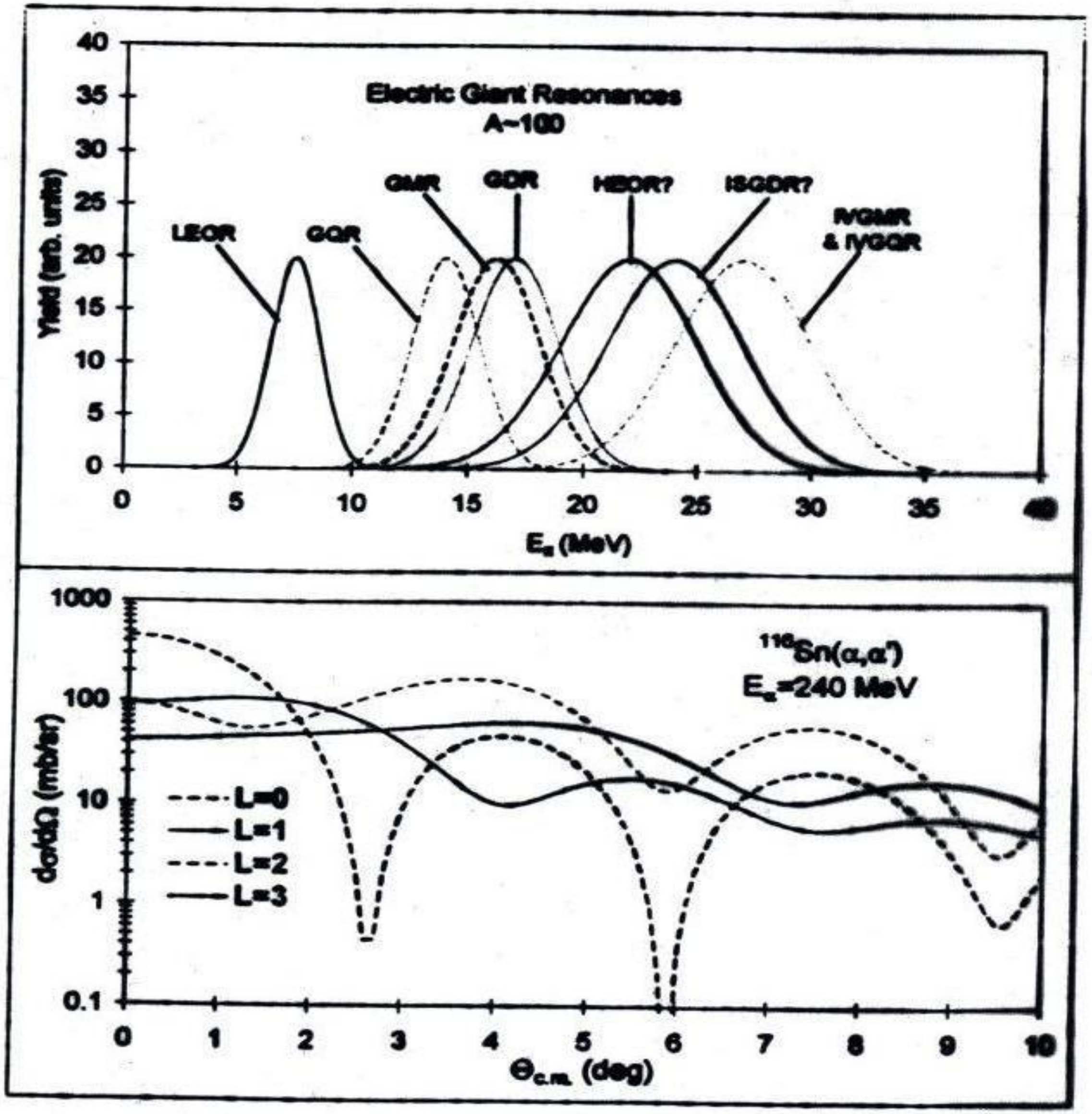
where the parameters α are determined by fit to data and compared to values consistent with 100% EWSR exhaustion.

EWSR = energy weighted sum rule

$$\left[\int_0^\infty ES(E)dE\right]$$



Elastic angular distributions for 240 MeV alpha particle. Filled squares represent the experimental data. Solid lines are fit to the experimental data using the folding model DWBA with nucleon-alpha interaction.



Since α particles have S = 0, T = 0, they are ideal for studying electric (ΔS = 0) and isoscalar (ΔT = 0) Giant Resonances.

History

A. ISOSCALAR GIANT MONOPOLE RESONANCE (ISGMR):

1977 – DISCOVERY OF THE CENTROID ENERGY OF THE ISGMR IN ^{208}Pb

$$E_0 \sim 13.5 \text{ MeV (TAMU)}$$

- This led to modification of commonly used effective nucleon-nucleon interactions. Hartree-Fock (HF) plus Random Phase Approximation (RPA) calculations, with effective interactions (Skyrme and others) which reproduce data on masses, radii and the ISGMR energies have:

$$K_\infty = 210 \pm 20 \text{ MeV (J.P. BLAIZOT, 1980).}$$

A. ISOSCALAR GIANT DIPOLE RESONANCE (ISGDR):

1980 – EXPERIMENTAL CENTROID ENERGY IN ^{208}Pb AT

$E_1 \sim 21.3$ MeV (Jülich), PRL 45 (1980) 337; ~ 19 MeV, PRC 63 (2001) 031301

- HF-RPA with interactions reproducing E_0 predicted $E_1 \sim 25$ MeV.

$$K_\infty \sim 170 \text{ MeV from ISGDR ?}$$

T.S. Dimitrescu and F.E. Serr [PRC 27 (1983) 211] pointed out “If further measurement confirm the value of 21.3 MeV for this mode, the discrepancy may be significant”.

→ Relativistic mean field (RMF) plus RPA with NL3 interaction predict K_∞=270 MeV from the ISGMR [N. Van Giai et al., NPA 687 (2001) 449].

Hartree-Fock (HF) - Random Phase Approximation (RPA)

In fully self-consistent calculations:

1. Assume a form for the Skyrme parametrization (δ-type).

2. Carry out HF calculations for ground states and determine the Skyrme parameters by a fit to binding energies and radii.

3. Determine the residual p-h interaction $V_{\mathrm{php'h'}} = \dfrac{\delta^2 E[\rho]}{\delta\rho_{\mathrm{ph}}\delta\rho_{\mathrm{p'h'}}}$

4. Carry out RPA calculations of strength function, transition density etc.

Green's Function Formulation of RPA

In the Green's Function formulation of RPA, one starts with the RPA-Green's function which is given by

$$G = G_o(1 + V_{ph}G_o)^{-1}$$

where V_{ph} is the particle-hole interaction and the free particle-hole Green's function is defined as

$$G_o(\mathbf{r}, \mathbf{r}', E) = -\sum_i \varphi_i{}^*(\mathbf{r})\left[\frac{1}{h_o - \varepsilon_i - E} + \frac{1}{h_o - \varepsilon_i + E}\right]\varphi_i(\mathbf{r}')$$

where φ_i is the single-particle wave function, ϵ_i is the single-particle energy, and h_o is the single-particle Hamiltonian.

The continuum effects, such as particle escape width, can be taken into account using

$$\left\langle r_1 \left| \frac{1}{h_0 - Z} \right| r_2 \right\rangle = \frac{2m}{\hbar^2} U(r_<) V(r_>) / W$$

where $r_<$ and $r_>$ are the lesser and greater of r_1 and r_2 respectively, U and V are the regular and irregular solution of $(H_0\text{-}Z)\psi = 0$, with the appropriate boundary conditions, and W is the Wronskian.

NOTE the two terms in the free particle-hole greens function

We use the scattering operator *F*

$$F = \sum_{i=1}^{A} f(\mathbf{r}_i)$$

to obtain the strength function

$$S(E) = \sum_n \left|\langle 0|F|n\rangle\right|^2 \delta(E - E_n) = \frac{1}{\pi}\mathrm{Im}[Tr(f \cdot G \cdot f)]$$

and the transition density

$$\delta\rho^{RPA} = \rho_t(\mathbf{r}, E) = \frac{\Delta E}{\sqrt{S(E)\cdot\Delta E}} \cdot \int f(\mathbf{r}')\cdot[\frac{1}{\pi}\mathrm{Im}\,G(\mathbf{r},\mathbf{r}',E)]d^3\mathbf{r}'$$

$\delta\rho^{RPA}$ is consistent with the strength in $E \pm \Delta E/2$

$$S(E) = \left|\int \delta\rho^{RPA}(r,E)f(r)d\vec{r}\right|^2 \Big/ \Delta E$$

Relativistic Mean Field + Random Phase Approximation

The steps involved in the relativistic mean field based RPA calculations are analogous to those for the non-relativistic HF-RPA approach. The nucleon-nucleon interaction is generated through the exchange of various effective mesons. An effective Lagrangian which represents a system of interacting nucleons looks like

$$\begin{aligned}\mathcal{L} = \overline{\Psi}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - M_N - g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu} - g_{\rho}\tau^{a}\gamma^{\mu}\rho^{a}_{\mu} - e\gamma^{\mu}A_{\mu}\tfrac{1}{2}(1-\tau_3))\Psi \\ + \tfrac{1}{2}\partial^{\mu}\sigma\,\partial_{\mu}\sigma - U_{\sigma} - \tfrac{1}{4}W^{\mu\nu}W_{\mu\nu} + U_{\omega} + \tfrac{1}{2}m_{\rho}^{2}\rho^{a\mu}\rho^{a}_{\mu} - \tfrac{1}{4}R^{a\mu\nu}R^{a}_{\mu\nu} \\ - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu},\end{aligned}$$

$$U_{\sigma} = \tfrac{1}{2}m_{\sigma}^{2}\sigma^{2} + \tfrac{1}{3}g_{2}\sigma^{3} + \tfrac{1}{4}g_{3}\sigma^{4}, \qquad U_{\omega} = \tfrac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu} + \tfrac{1}{4}c_{3}(\omega^{\mu}\omega_{\mu})^{2}.$$

It contains nucleons (ψ) with mass M; σ, ω, ρ mesons; the electromagnetic field; non linear self-interactions for the σ (and possibly ω) field.

Values of the parameters for the most widely used NL3 interaction are m_σ=508.194 MeV, m_ω=782.501 MeV, m_ρ=763.000 MeV, g_σ=10.217, g_ω=12.868, g_ρ=4.474, g_2=-10.431 fm^{-1} and g_3=-28.885 (in this case there is no self-interaction for the ω meson).

NL3: K_∞=271.76 MeV, G.A.Lalazissis et al., PRC 55 (1997) 540.

RMF-RPA: J. Piekarewicz PRC 62 (2000) 051304; Z.Y. Ma et al., NPA 686 (2001) 173.

Self-consistent calculation within constrained HF

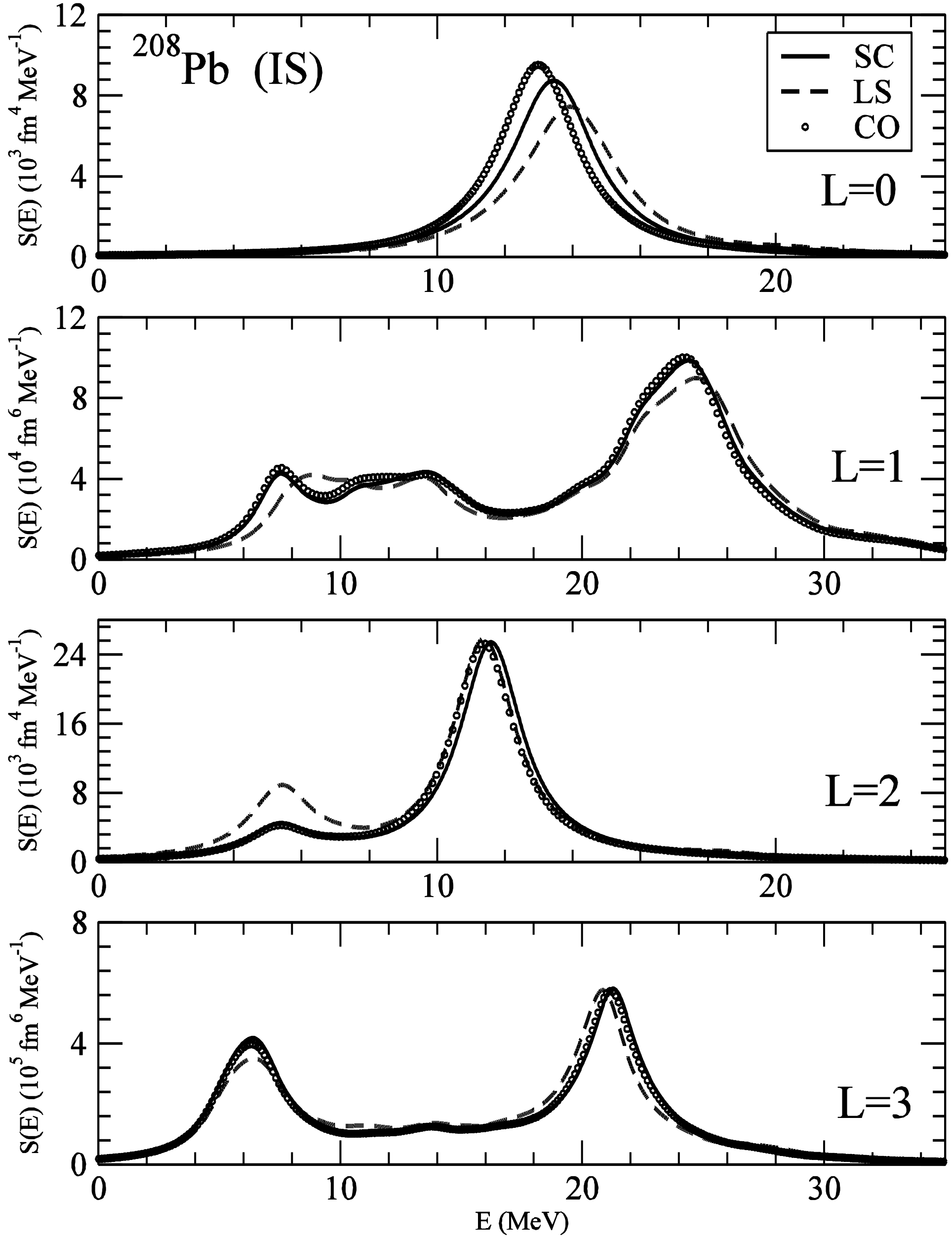
$H_\lambda = H - \lambda f$ monopole $f = \sum_{i=1}^{A} r_i^2$

$E_{constr.} = (m_1/m_{-1})^{1/2}$ $E_s = (m_3/m_1)^{1/2}$

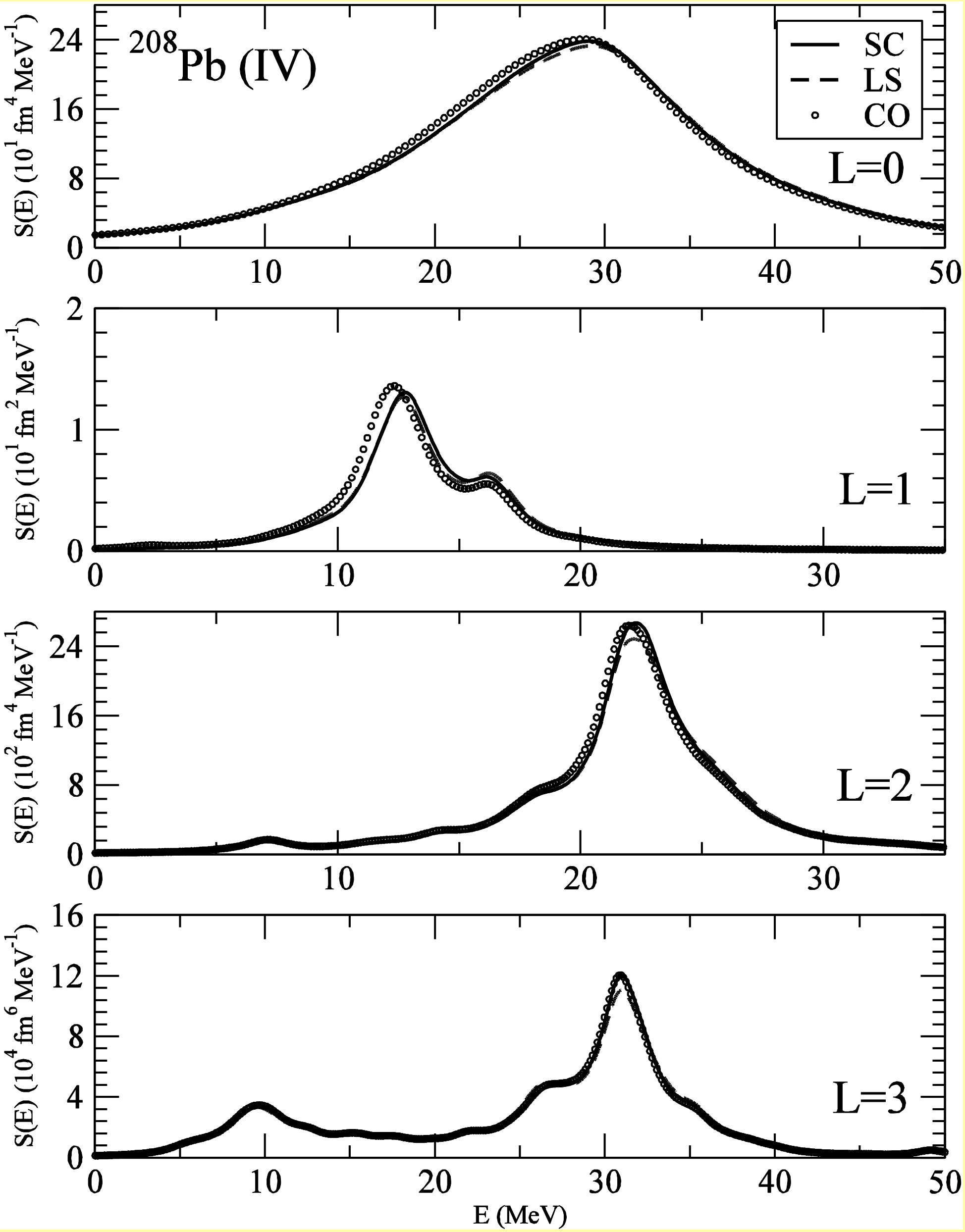
$m_k = \int E^k S(E) dE$ $m_1 = 2\frac{\hbar}{m}\langle r^2\rangle$

$m_{-1} = \frac{1}{2}\frac{d}{d\lambda}\langle r_\lambda^2\rangle\Big|_{\lambda=0} = \frac{1}{2}\frac{d^2}{d\lambda^2}\langle H_\lambda\rangle\Big|_{\lambda=0}$

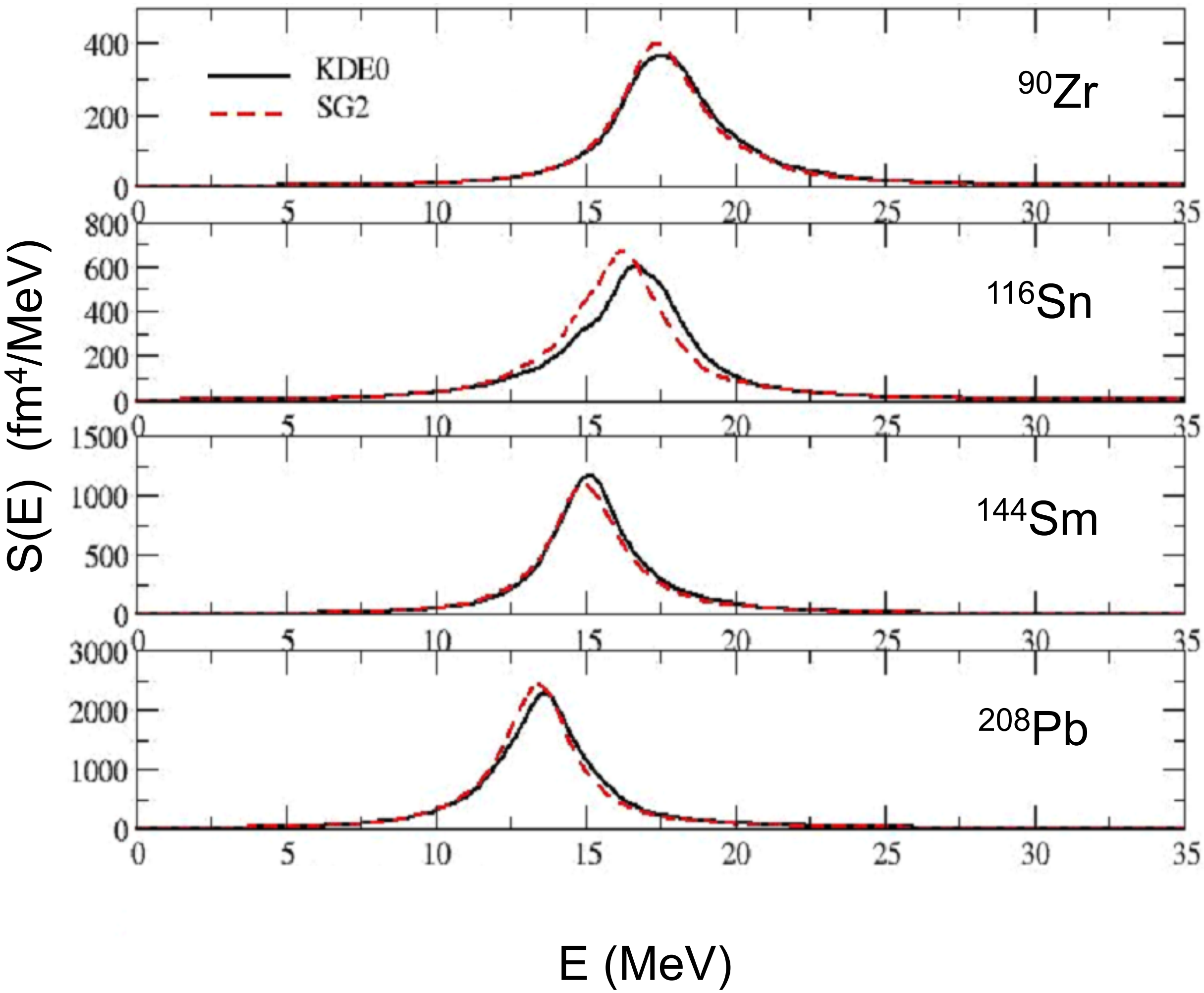
$m_3 = \frac{1}{2}(\frac{2\hbar}{m})^2[2T + E_\delta + 20(E_{fin} + E_{s.o.}) + (3\alpha + 2)(3\alpha + 3)E_\rho]$



Isoscalar strength functions of ^{208}Pb for L = 0 - 3 multipolarities are displayed. The SC (full line) corresponds to the fully self-consistent calculation where LS (dashed line) and CO (open circle) represent the calculations without the ph spin-orbit and Coulomb interaction in the RPA, respectively. The Skyrme interaction SGII [Phys. Lett. B **106**, 379 (1981)] was used.



Isovector strength functions of ^{208}Pb for L = 0 - 3 multipolarities are displayed. SC (full line) corresponds to the fully self-consistent calculation where LS (dashed line) and CO (open circle) represent the calculations without the ph spin-orbit and Coulomb interaction in the RPA, respectively. The Skyrme interaction SGII [Phys. Lett. B **106**, 379 (1981)] was used.

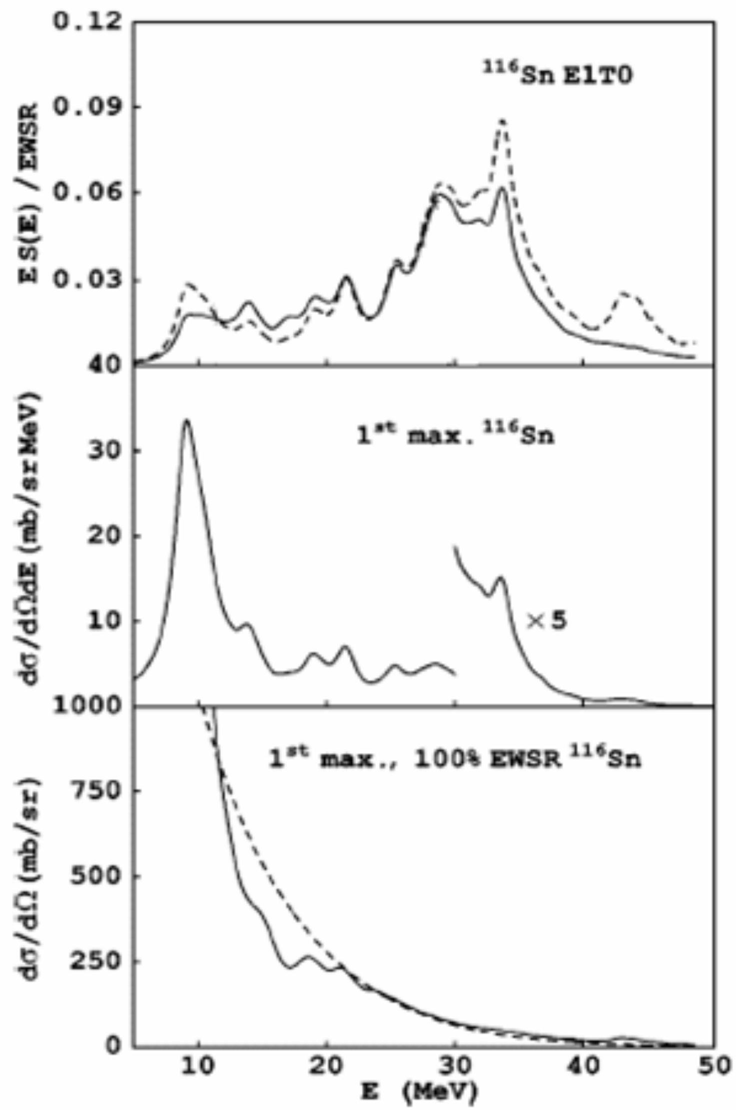


Isoscalar monopole strength function

S. Shlomo and A.I. Sanzhur, Phys. Rev. C **65**, 044310 (2002)

ISGDR $f = \left(r^3 - \frac{5}{3}\langle r^2 \rangle r \right) Y_{1M}$

SL1 interaction, K = 230 MeV, E_α = 240 MeV

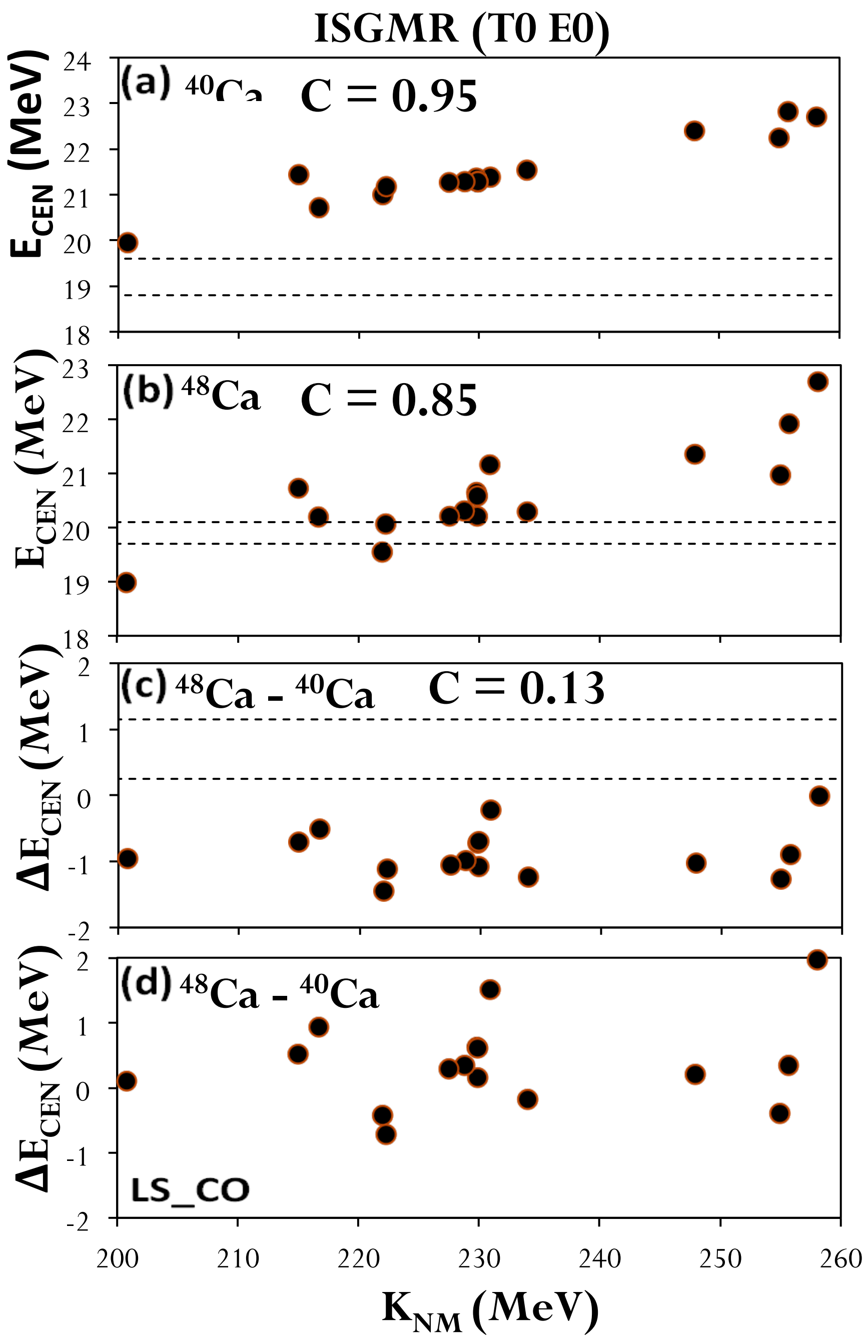


Reconstruction of the ISGDR EWSR in ^{116}Sn from the inelastic α-particle cross sections. The middle panel: maximum double differential cross section obtained from ρ_t (RPA). The lower panel: maximum cross section obtained with ρ_{coll} (dashed line) and ρ_t (solid line) normalized to 100% of the EWSR. Upper panel: The solid line (calculated using RPA) and the dashed line are the ratios of the middle panel curve with the solid and dashed lines of the lower panel, respectively.

$$\rho_{coll} = \left[10r + \left(3r^2 - \frac{5}{3}\langle r^2 \rangle \right) \frac{d\rho_0}{dr} \right] \rho_0(r)$$

Fully self-consistent HF-RPA results for ISGMR centroid energy (in MeV) with the Skyrme interaction SK255, SGII and KDE0 are compared with the RRPA results using the NL3 interaction. Note the corresponding values of the nuclear matter incompressibility, K, and the symmetry energy , J, coefficients. ω_1-ω_2 is the range of excitation energy. The experimental data are from TAMU.

Nucleus	ω_1-ω_2	Expt.	NL3	SK255	SGII	KDE0
^{90}Zr	0-60		18.7	18.9	17.9	18.0
	10-35	17.81±0.30		18.9	17.9	18.0
^{116}Sn	0-60		17.1	17.3	16.4	16.6
	10-35	15.85±0.20		17.3	16.4	16.6
^{144}Sm	0-60		16.1	16.2	15.3	15.5
	10-35	15.40±0.40		16.2	15.2	15.5
^{208}Pb	0-60		14.2	14.3	13.6	13.8
	10-35	13.96±0.30		14.4	13.6	13.8
K (MeV)			272	255	215	229
J (MeV)			37.4	37.4	26.8	33.0



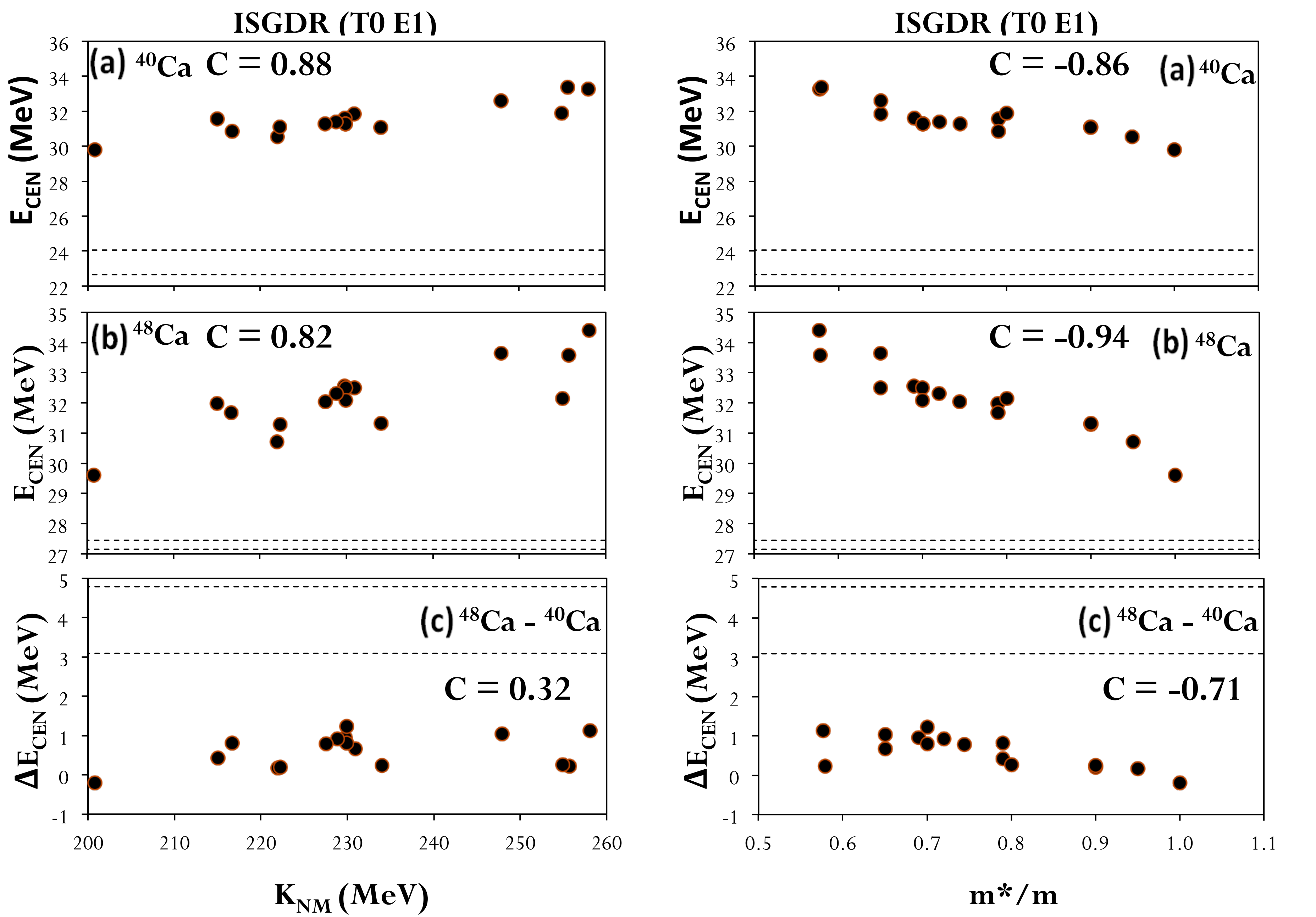
None of the interactions fall in the Experimental range for ^{40}Ca

$$E_{CEN} = \frac{\int ES(E)dE}{\int S(E)dE}$$

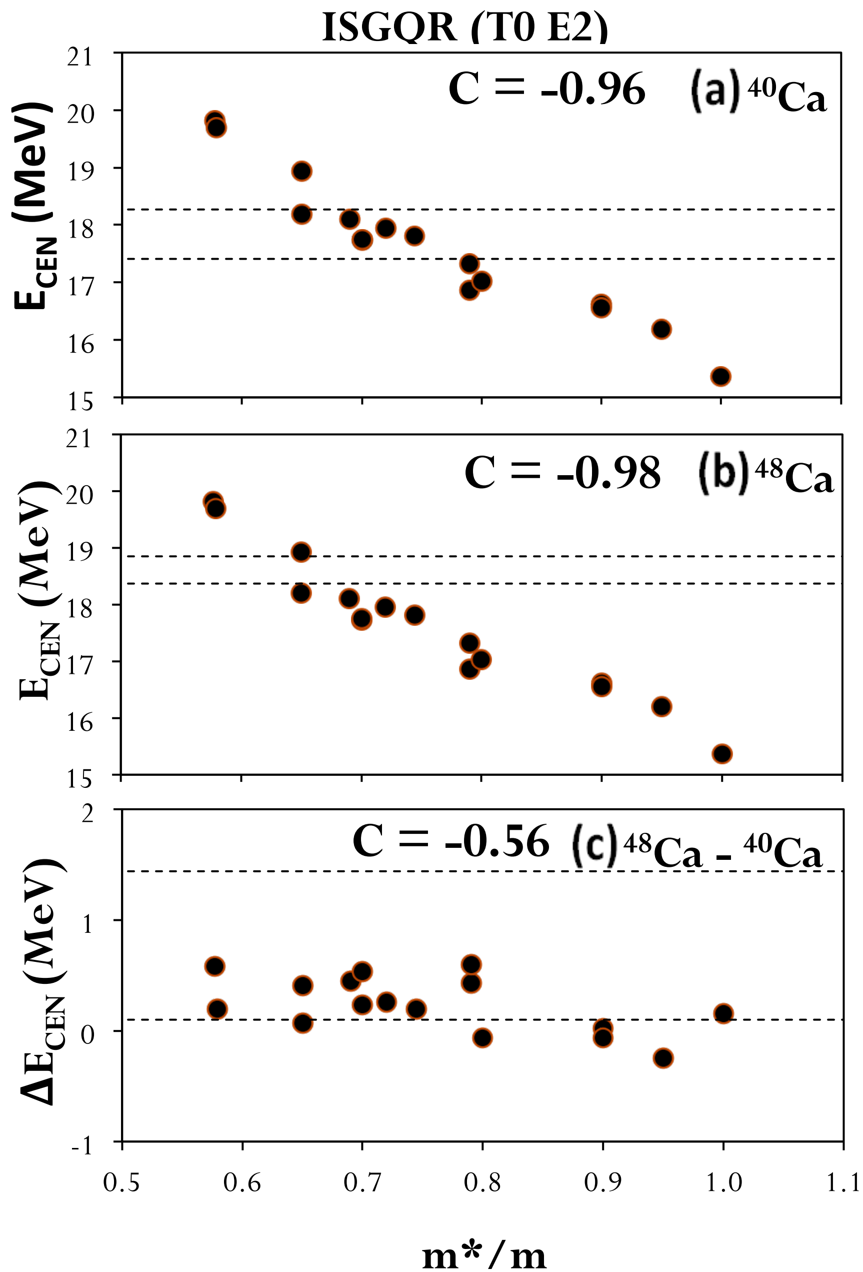
C is the Pearson correlation coefficient

^{48}Ca – ^{40}Ca > 0 for all the interactions which goes against the trend of decreasing ISGMR with increasing A

Note that for not self-consistent RPA calculations, which neglect the Coulomb and Spin-Orbit parts. Some interactions would fall in the correct ^{48}Ca – ^{40}Ca range.



ISGDR (T0 E1)
ISGDR (T0 E1)
(a) 40Ca C = 0.88
C = -0.86 (a) 40Ca
(b) 48Ca C = 0.82
C = -0.94 (b) 48Ca
(c) 48Ca - 40Ca
C = 0.32
C = -0.71
ECEN (MeV)
ΔECEN (MeV)
KNM (MeV)
m*/m



ISGQR (T0 E2)
C = -0.96
(a) 40Ca
C = -0.98
(b) 48Ca
C = -0.56
(c) 48Ca - 40Ca
E_CEN (MeV)
ΔE_CEN (MeV)
21
20
19
18
17
16
15
2
1
0
-1
0.5
0.6
0.7
0.8
0.9
1.0
1.1
m*/m

The Equation of State of neutron-rich nuclear matter

symmetry energy

isospin asymmetry $\delta=(\rho_n-\rho_p)/\rho$

$$E(\rho_n,\rho_p)\ ;\ E_0(\rho_n=\rho_p)+E_{sym}(\rho)\delta^2+O(\delta^4)$$

EOS of isospin-symmetric nuclear matter
with equal numbers of protons and neutrons

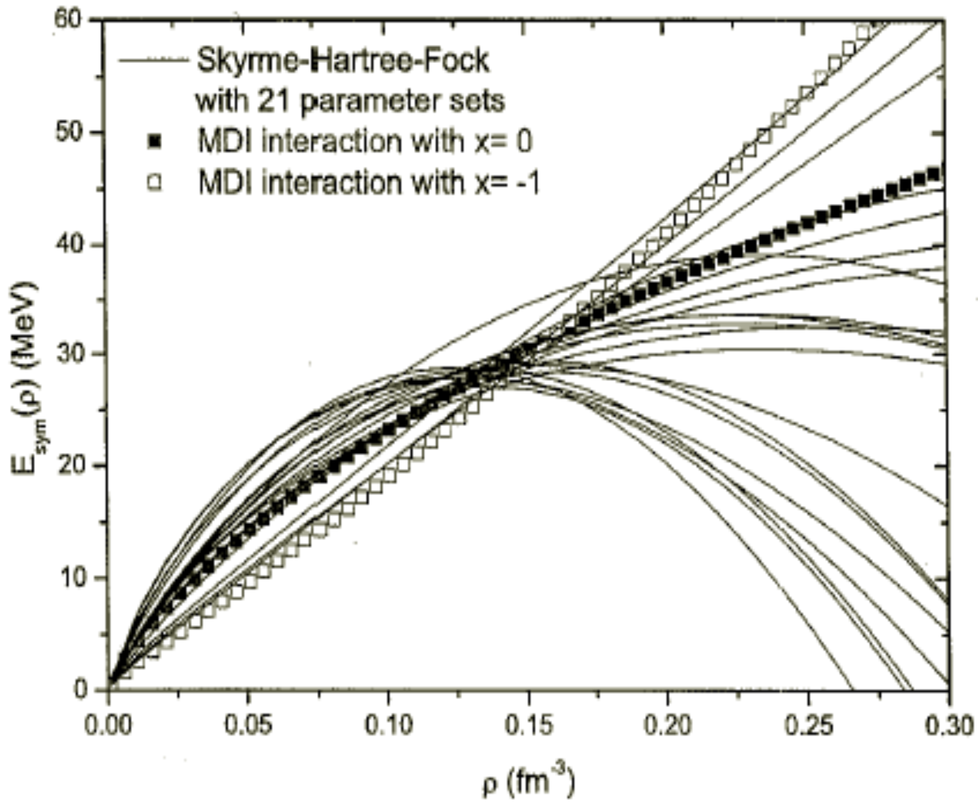
Energy per nucleon in isospin-asymmetric nuclear matter
with different numbers of protons and neutrons

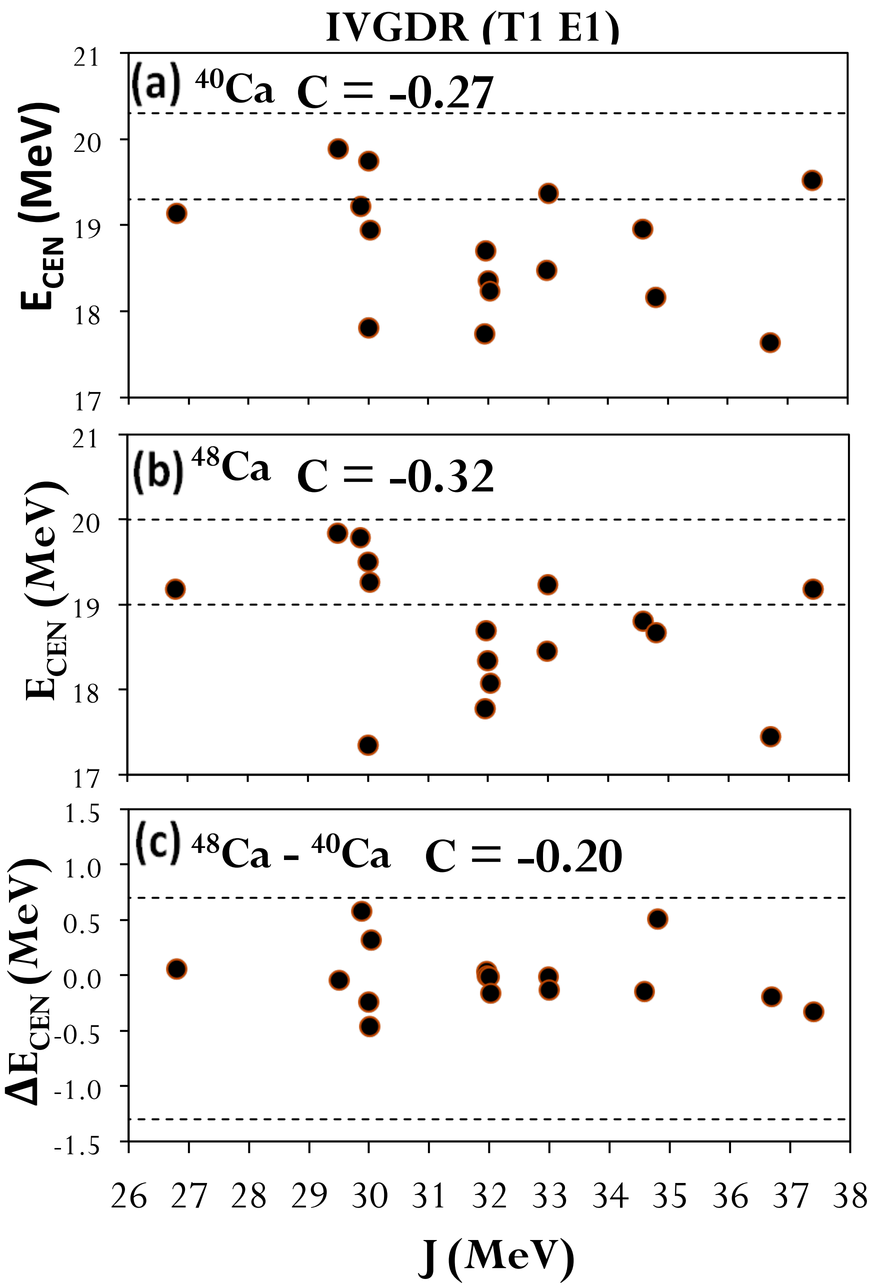
- This parabolic approximation is valid up to pure neutron matter as shown by all existing many-body theories

- The EOS of isospin-symmetric nuclear matter is relatively well determined after almost 30 years of hard work by many people in the nuclear physics community

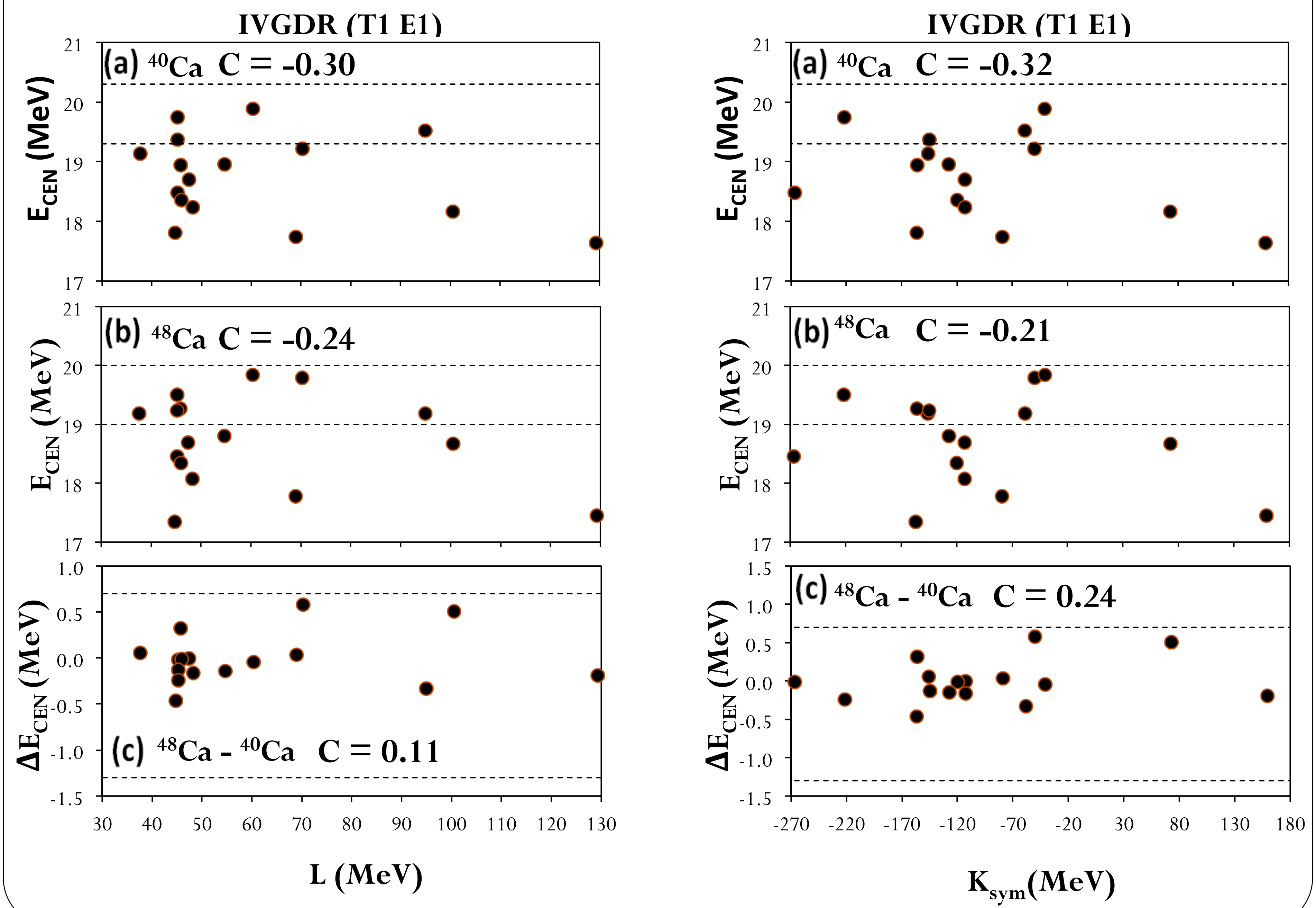
- Besides the possible phase transition at high densities, the symmetry energy $E_{sym}(\rho)$ is the most uncertain term in the EOS of neutron-rich matter

Constraining nuclear effective interactions within Skyrme Hartree-Fock

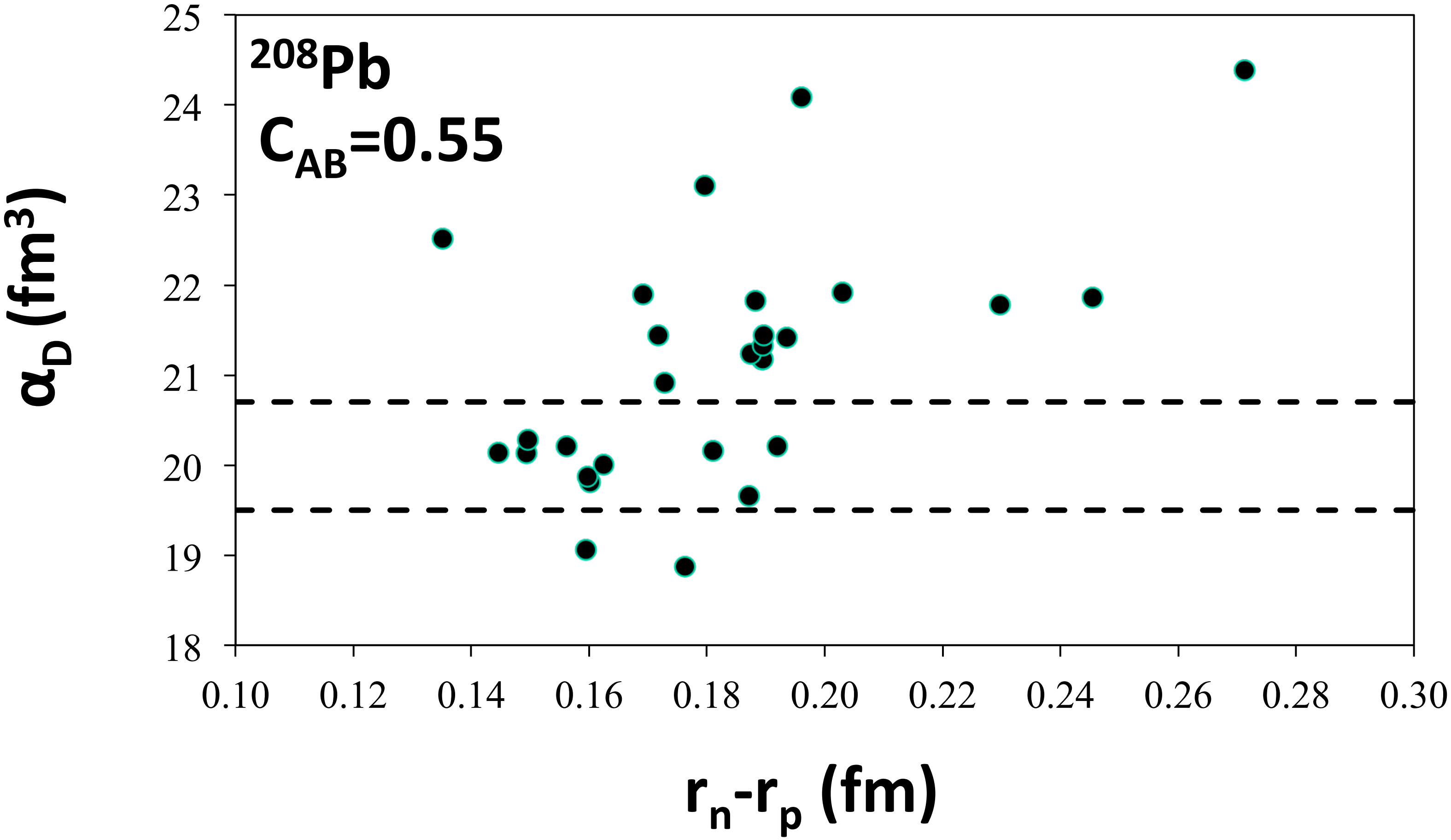




No clear value of the symmetry energy, J can be deduced.



IVGDR (T1 E1)
(a) 40Ca C = -0.30
(b) 48Ca C = -0.24
(c) 48Ca - 40Ca C = 0.11
ECEN (MeV)
ΔECEN (MeV)
L (MeV)
IVGDR (T1 E1)
(a) 40Ca C = -0.32
(b) 48Ca C = -0.21
(c) 48Ca - 40Ca C = 0.24
Ksym(MeV)



208Pb
CAB=0.55
αD (fm3)
rn-rp (fm)
25
24
23
22
21
20
19
18
0.10
0.12
0.14
0.16
0.18
0.20
0.22
0.24
0.26
0.28
0.30

Conclusions

- We have developed a new EDFs based on Skyrme type interaction (KDE0, KDE, KDE0v1,...) applicable to properties of rare nuclei and neutron stars.
- Fully self-consistent calculations of the compression modes (ISGMR and ISGDR) within HF-based RPA using Skyrme forces and within relativistic model lead a nuclear matter incompressibility coefficient of $K_\infty = 240 \pm 20$ MeV, sensitivity to symmetry energy.
- Sensetivity to symmetry energy: IVGDR, GR in neutron rich nuclei, Rn – Rp, stll open problems.
- Possible improvements:
  - Account for effect of correlations on B.E. Radii, S.P. energies
  - Properly account for the isospin dependency of the spin-orbit interaction
  - Include additional data, such as IVGDR (J) and ISGQR (m*)

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CONCLUSION

We have developed a new EDF based on Skyrme type interaction (KDE0) applicable to properties of rare nuclei and neutron stars.

Fully self-consistent calculations of the ISGMR using Skyrme forces lead a nuclear matter incompressibility coefficient of $K_\infty = 240 \pm 20$ MeV with sensitivity to symmetry energy.

It is possible to build *bonafide* Skyrme forces with K close to the relativistic value.

Further improvement

(i) Account for the effect of correlation on B.E., R_{ch} and single particle energy

(ii) Properly account for the isospin dependency of the spin-orbit interaction

(iii) Include additional data, such as IVGDR (J) and ISGQR (m*)

SUMMARY AND CONCLUSIONS

1) Fully self-consistent calculations of the compression modes (ISGMR and ISGDR) using modern energy density functionals (Skyrme forces) lead to → K_∞ = 240 ± 20 MeV, with sensitivity to symmetry energy.

   Symmetry energy density (IVGDR, Rn – Rp, ...) --Open problem

2) Accounting for post-emission decay allows one to obtain consistent values of temperature of a disassembling source from the "double-ratio" method.

3) Although , at low densities, the temperature calculated from given yields changes only modestly if medium effects are taken into account, larger discrepancies are observed when the nucleon densities are determined from measured yields,

4) Due to clusterization at low density nuclear matter, the symmetry energy is much larger than that predicted by mean field approximation

Outline

1. Introduction

Definitions: nuclear matter incompressibility coefficient K_∞

Background: isoscalar giant monopole resonance,

isoscalar giant dipole resonance

Hadron excitation of giant resonances

1. Theoretical approaches for giant resonances

Hartree-Fock plus Random Phase Approximation (RPA)

Comments: self-consistency ?

Relativistic mean field (RMF) plus RPA

1. Discussion

ISGMR vs. ISGDR

Non-relativistic viz. Relativistic

Nuclear matter properties from collective modes in nuclei

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New Skyrme effective nucleon-nucleon interaction

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Effects of self-consistence violations in HF-based RPA calculations for giant resonances

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Texas A&M University

Nuclear matter equation of state and giant resonances in nuclei

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Introduction

The effective Skyrme interaction has been used in mean-field models for several decades and many different parameterizations of the interaction have been realized to better reproduce nuclear masses, radii, and various other data. Today, there is more experimental data on nuclei far from the stability line. It is time to improve the parameters of Skyrme interactions. We fit our mean-field results to an extensive set of experimental data and obtain the parameters of the Skyrme type effective interaction for nuclei at and far from the stability line.

The total energy

$$E = \left\langle \Phi \middle| \hat{H}_{total} \middle| \Phi \right\rangle = \left\langle \Phi \middle| T + V_{Coulomb} + V_{12} \middle| \Phi \right\rangle = \int \left[H_{Kinetic}(\vec{r}) + H_{Coulomb}(\vec{r}) + H_{Skyrme}(\vec{r}) \right] d\vec{r}$$

Where $$H_{Kinetic}(\vec{r}) = \frac{\hbar^2}{2m_p}\tau_p(\vec{r}) + \frac{\hbar^2}{2m_n}\tau_n(\vec{r})$$

$$H_{Coulomb}(\vec{r}) = \frac{e^2}{2}\left[\rho_{ch}(\vec{r}) \int \frac{\rho_{ch}(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,' - \int \frac{|\rho_{ch}(\vec{r}, \vec{r}\,')|^2}{|\vec{r} - \vec{r}\,'|} d\vec{r}\,' \right]$$

$$H_{Skyrme}(\vec{r}) = \frac{1}{2}t_0\left(1 + \frac{1}{2}x_0\right)\rho^2(\vec{r}) - \frac{1}{2}t_0\left(1 + \frac{1}{2}x_0\right)\left[\rho_p^2(\vec{r}) + \rho_n^2(\vec{r})\right] + \frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\rho(\vec{r})\tau(\vec{r})$$

$$- \frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) - t_2\left(1 + \frac{1}{2}x_2\right)\right]\left(\rho_p(\vec{r})\tau_p(\vec{r}) + \rho_n(\vec{r})\tau_n(\vec{r})\right) - \frac{1}{16}\left[3t_1\left(1 + \frac{1}{2}x_1\right) - t_2\left(1 + \frac{1}{2}x_2\right)\right]\rho(\vec{r})\nabla^2\rho(\vec{r})$$

$$+ \frac{1}{16}\left[3t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\left(\rho_p(\vec{r})\nabla^2\rho_p(\vec{r}) + \rho_n(\vec{r})\nabla^2\rho_n(\vec{r})\right) + \frac{1}{16}\left[t_1 - t_2\right]\left(\vec{J}_p^2(\vec{r}) + \vec{J}_n^2(\vec{r})\right) - \frac{1}{16}\left[t_1x_1 + t_2x_2\right]\vec{J}^2(\vec{r})$$

$$- \frac{1}{2}W_0\left[\rho(r)\vec{\nabla}\vec{J}(\vec{r}) + \rho_p(r)\vec{\nabla}\vec{J}_p(\vec{r}) + \rho_n(r)\vec{\nabla}\vec{J}_n(\vec{r}))\right] + \frac{1}{12}t_3\left[\rho^{\alpha+2}(\vec{r})\left(1 + \frac{1}{2}x_3\right) - \rho^{\alpha}(\vec{r})\left(\rho_p^2(\vec{r}) + \rho_n^2(\vec{r})\right)\left(x_3 + \frac{1}{2}\right)\right]$$

MODERN ENERGY DENSITY FUNCTIONAL FOR NUCLEI AND THE NUCLEAR MATTER EQUATION OF STATE

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