One-Dimensional Kinematics

One dimensional kinematics refers to motion along a straight line.

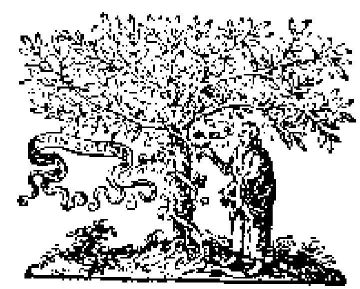
- Even though we live in a 3-dimension world, motion can often be abstracted to a single dimension.
- We can also describe motion along a curved path as one-dimensional motion.

Terms we will use:

- Position, distance, displacement
- Speed, velocity (average and instantaneous)
- Acceleration (average and instantaneous)

Galileo (1564-1642)

DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con una Appendice del centro di grauità d'alcuni Solidi.



IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.

What mathematical forces govern accelerated motion?

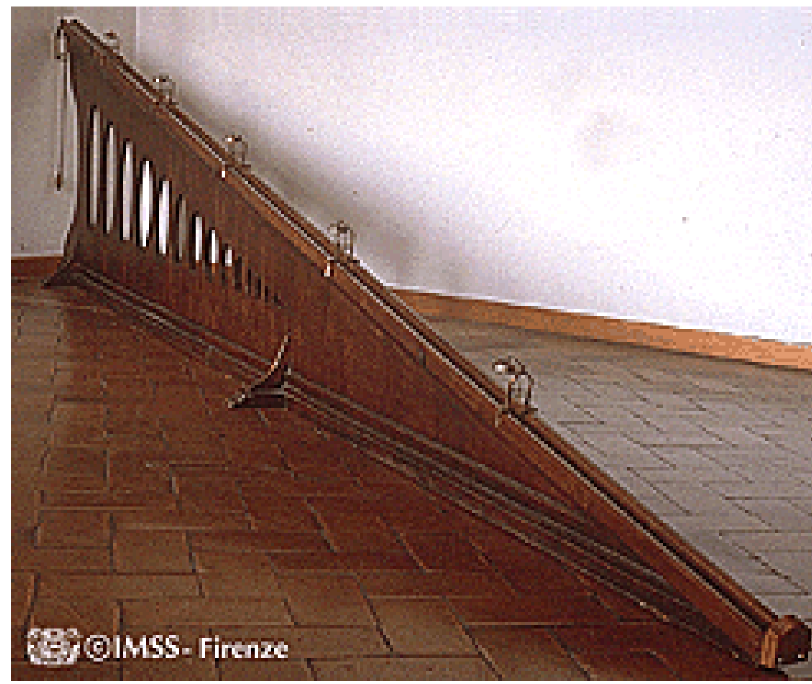
A wooden and stone sphere are dropped from the tower of pisa. Which one reaches the earth first?

Answer: same time!

The inclined plane experiment

Distance traveled goes with the square of time:

$x(t) \sim t^2$



Time measured with water!

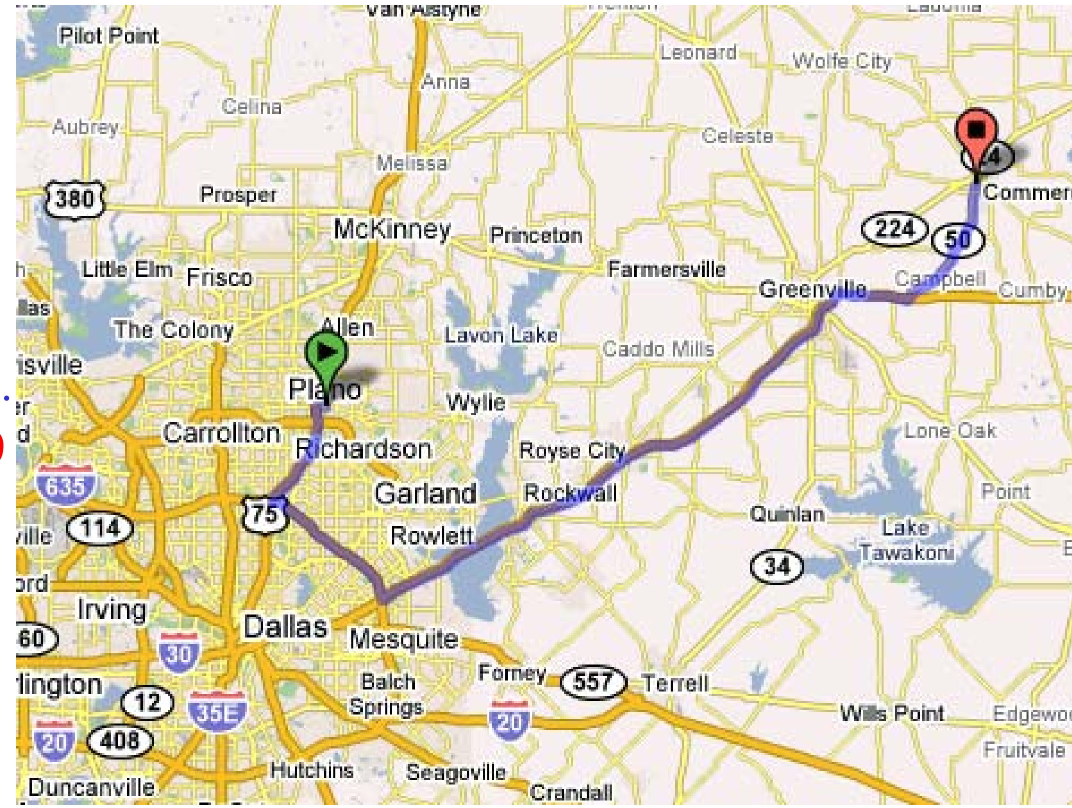
Coordinates & Vectors

Not only do we always use a quantitative measure for distance and time, we also intuitively use vectors and coordinates for describing locations and displacements.

For example, give directions from Texas A&M Commece to Plano, TX.

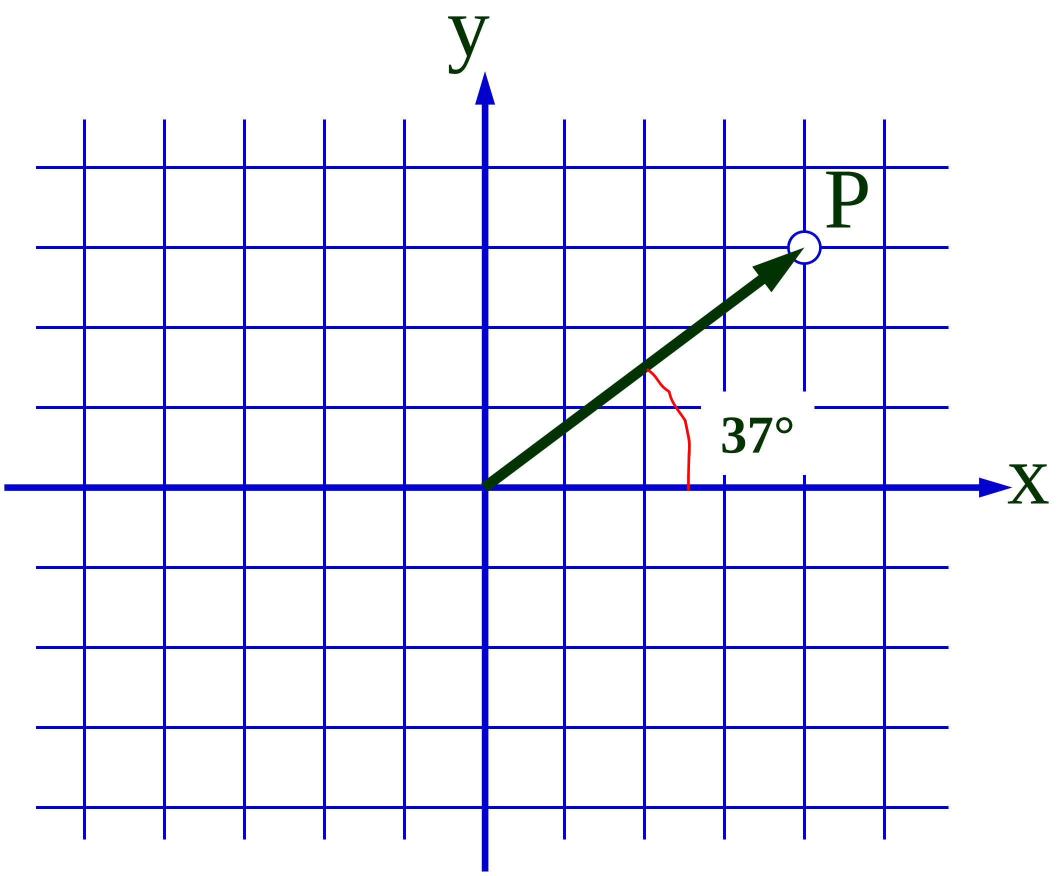
- Turn LEFT onto TX-24 S / TX-50 S. 9.8 miles.
- Merge onto I-30 W / US-67 S. 54.4 miles.
- Take the US-75 N exit- EXIT 47B- toward SHERMAN. 0.5 miles.
- Take the MAIN ST WEST / ELM ST exit on the LEFT. 0.2 miles.

Each instruction includes both distance and direction



Vectors

- To define a position in (2-dim) space, we define an origin and a coordinate system (x,y), (N,E) etc.
- A point P can be described as an ordered pair (x,y) = (4m,3m) or a vector of length 5 m pointing 36.8° counter-clockwise from the x-axis.
- We can think of the point P as either a position, or a displacement: P is 4 m along the x-axis plus 3 m along the y-axis



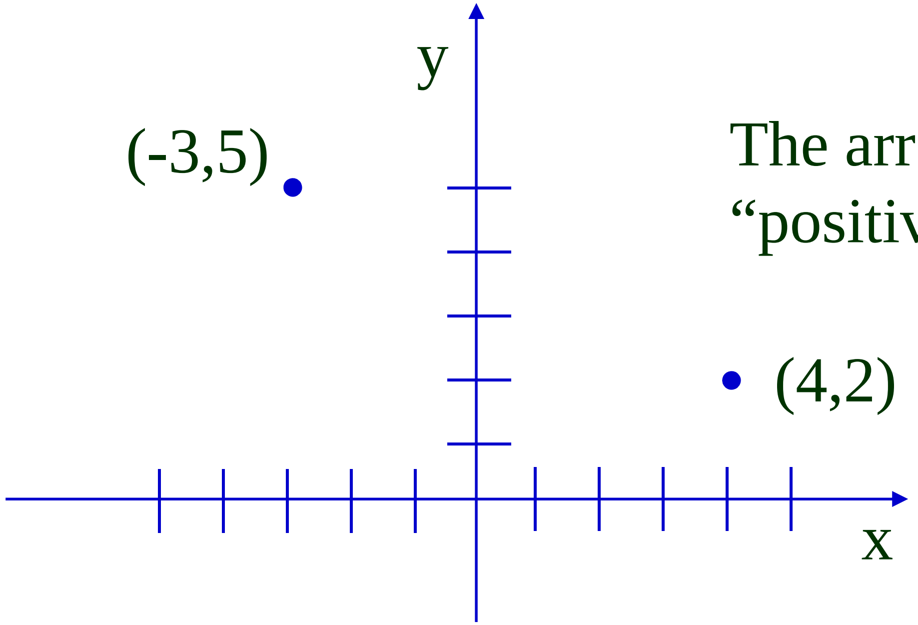
Coordinate Systems

A coordinate system is used to describe location.

A coordinate system consists of:

- a fixed reference point called the origin
- a set of axes
- a definition of the coordinate variables

Example



The arrow indicates the "positive" direction.

cartesian

The position of an object is its location in a coordinate system.

Distance and displacement

Distance is the total length of travel.

- It is always positive.
- It is measured by the odometer in your car.

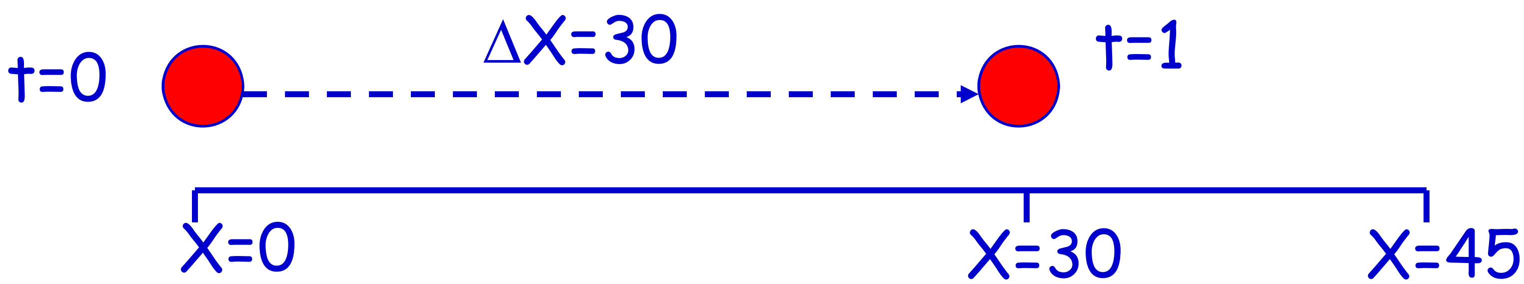
Displacement is defined as the change in position of an object.

$$\Delta x \equiv x_f - x_i$$

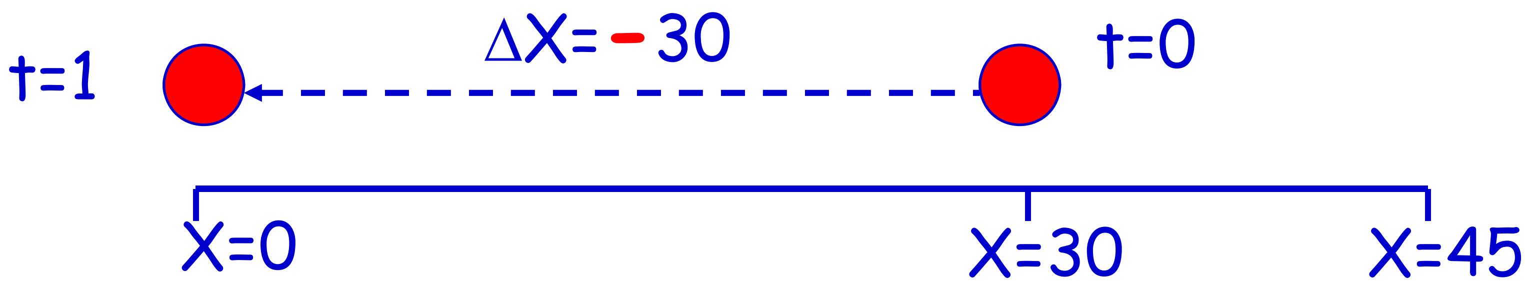
'Δ' (Delta)=change

- x_f = final value of x, x_i = initial value of x
- Change can be positive, negative or zero.
- Displacement is a vector (see Chapter 3)

Displacement



Displacement is a vector and thus has direction (sign)
Displacement is not equal to Distance



Average Speed and Velocity

Speed and velocity are not the same in physics!

Speed is **rate** of change of distance:

$$\text{average speed} = \frac{\text{distance}}{\text{elaspsed time}}$$ (always positive)

Velocity is **rate** of change of displacement:

$$\text{average velocity} = \frac{\text{displacement}}{\text{elaspsed time}} = \frac{x_f - x_i}{t_f - t_i}$$ (positive, negative or zero)

velocity is a vector (see Chapter 3)

Here we are just giving the 'x-component' of velocity, assuming the other components are either zero or irrelevant to our present discussion

SI units of speed and velocity are m/s.

Example

What is the average speed of a person at the equator due to the Earth's rotation?

Distance travelled in one day (one rotation) equals circumference of earth = (2π)(radius) = $(2\pi)(6.37 \bullet 10^6$ m) = $4.002 \bullet 10^7$ m = $4.002 \bullet 10^4$ km
Average speed = ($4.002 \bullet 10^4$ km)/(24hour) = $1.67 \bullet 10^3$ km/hr
You don't feel this! Velocity (in itself) is not important to dynamics!

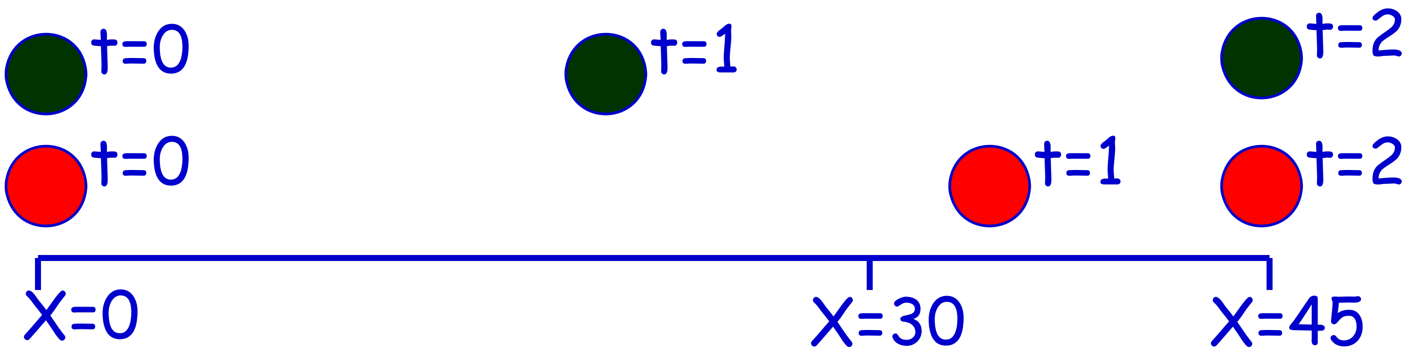
What is the average velocity of a person at the equator due to the Earth's rotation?

Zero

Average Velocity

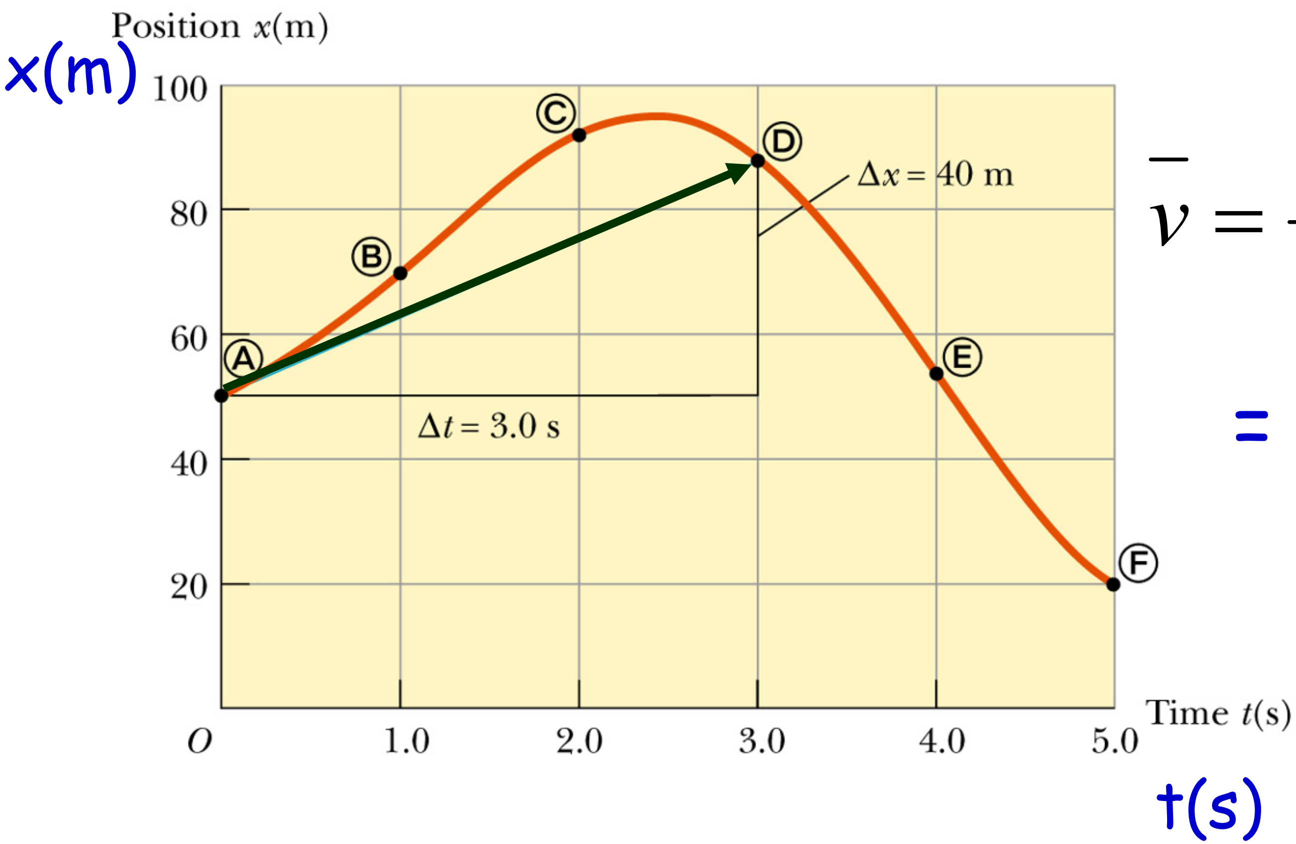
Average velocity= $\dfrac{\text{Displacement}}{\text{Time interval}}$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$



Which object has the largest average velocity over 2 s?

Graphically

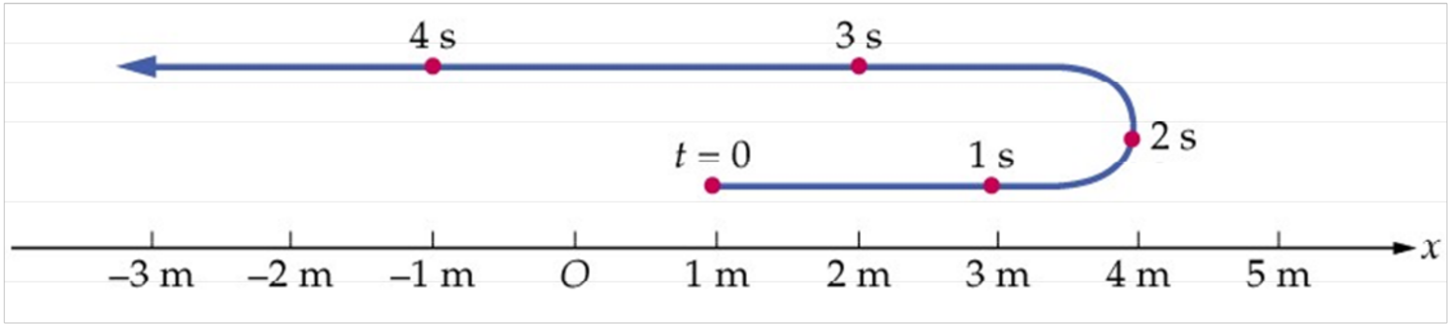


$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

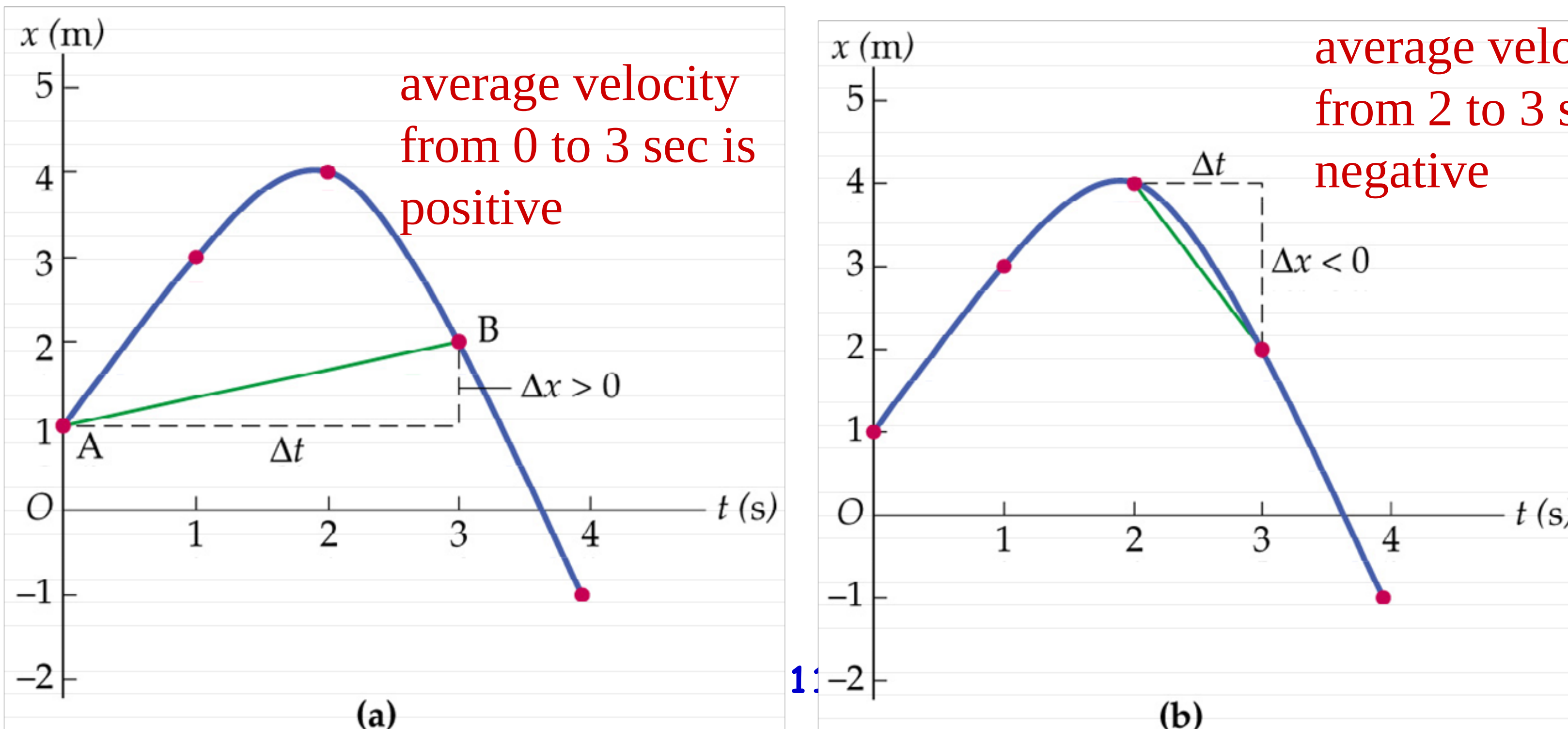
= slope

The average velocity is the slope of the line connecting begin and end point in the x-t graph. Ignore what happens in between.

Position vs. Time Plots



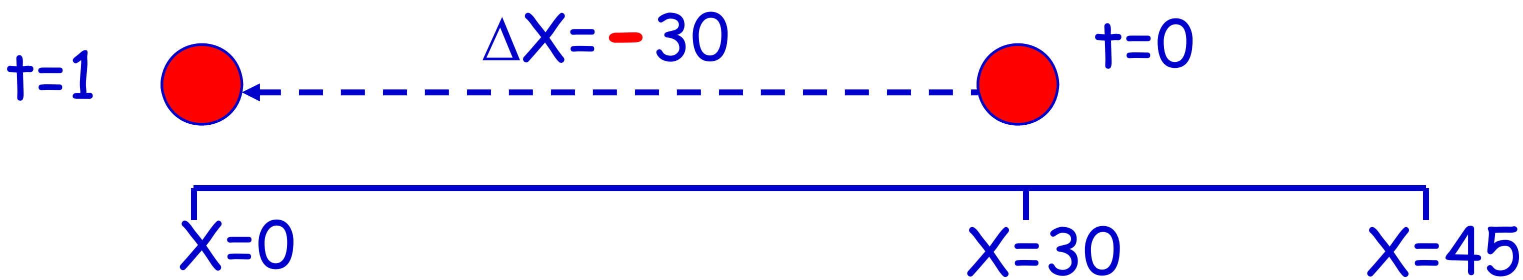
The average velocity between two times is the slope of the straight line connecting those two points.



average velocity from 0 to 3 sec is positive

average velocity from 2 to 3 sec is negative

Vectors and Scalars...



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$ =-30

Average velocity:vector

$$\overline{speed} = \frac{|\Delta x|}{\Delta t} = \frac{|x_f - x_i|}{t_f - t_i}$$ =+30

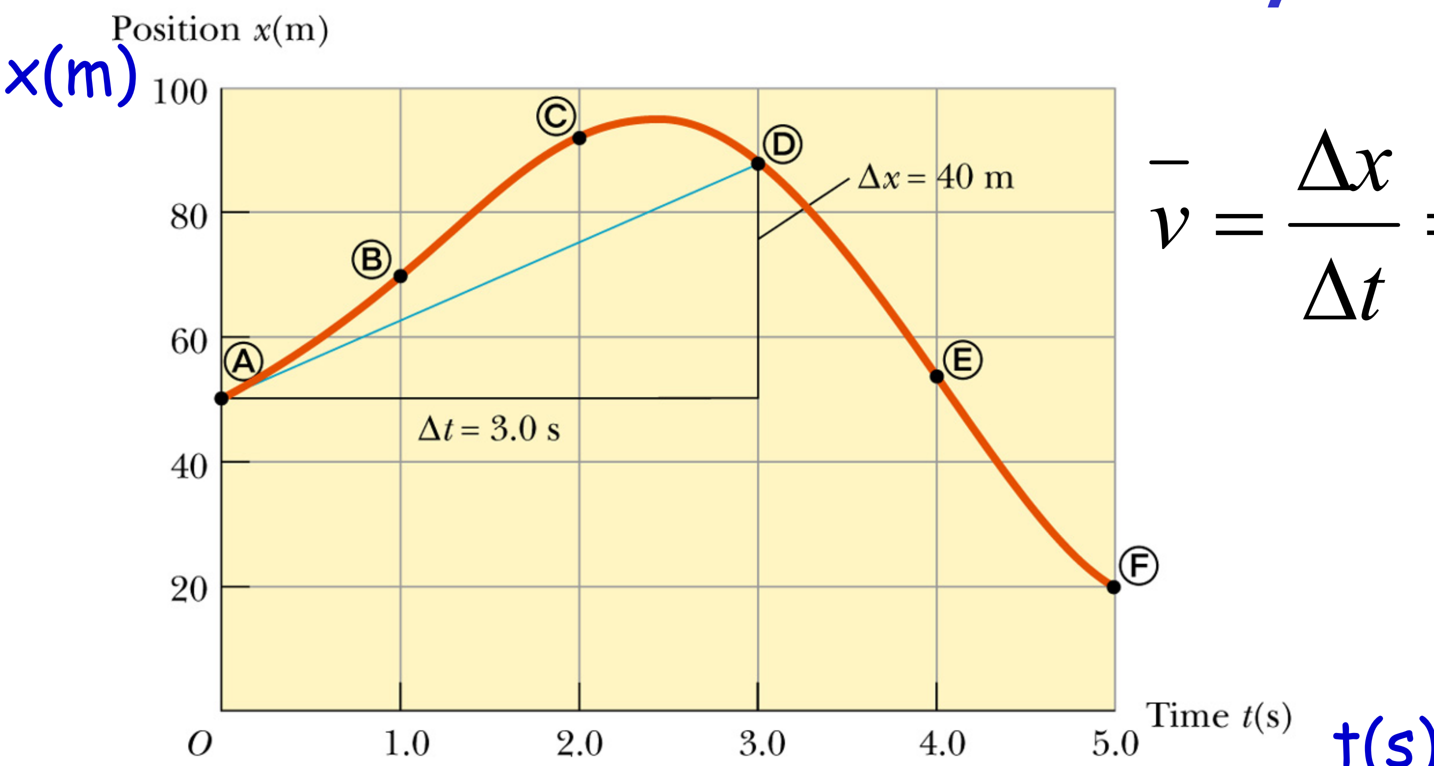
Average speed:scalar

Instantaneous Velocity

"But officer, I only drove 1 mile! How would you know I drove at 60 miles per hour…"

Sometimes we want to know the speed at one particular point in time.

Instantaneous velocity

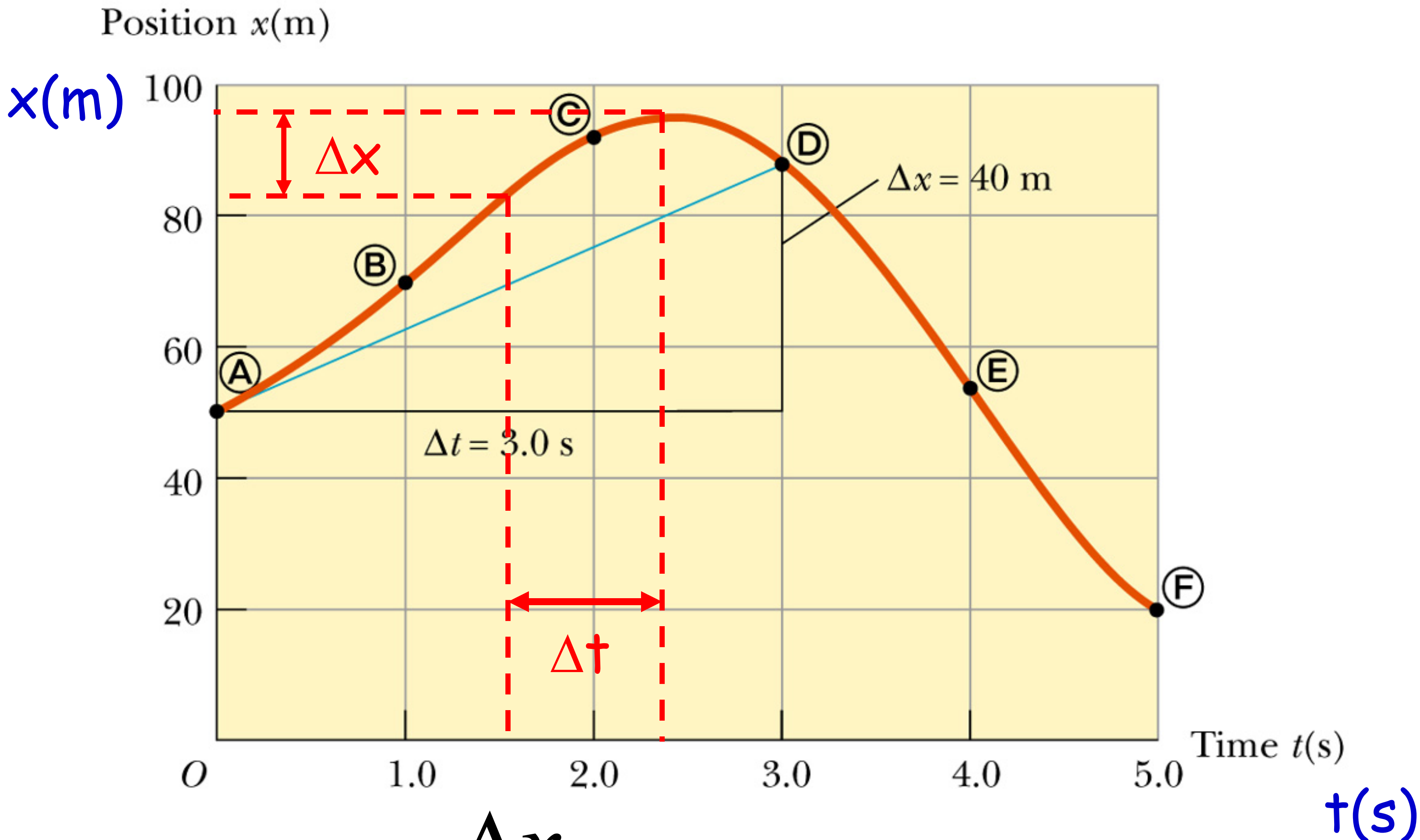


$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

What is the Velocity at t=2.0 s?

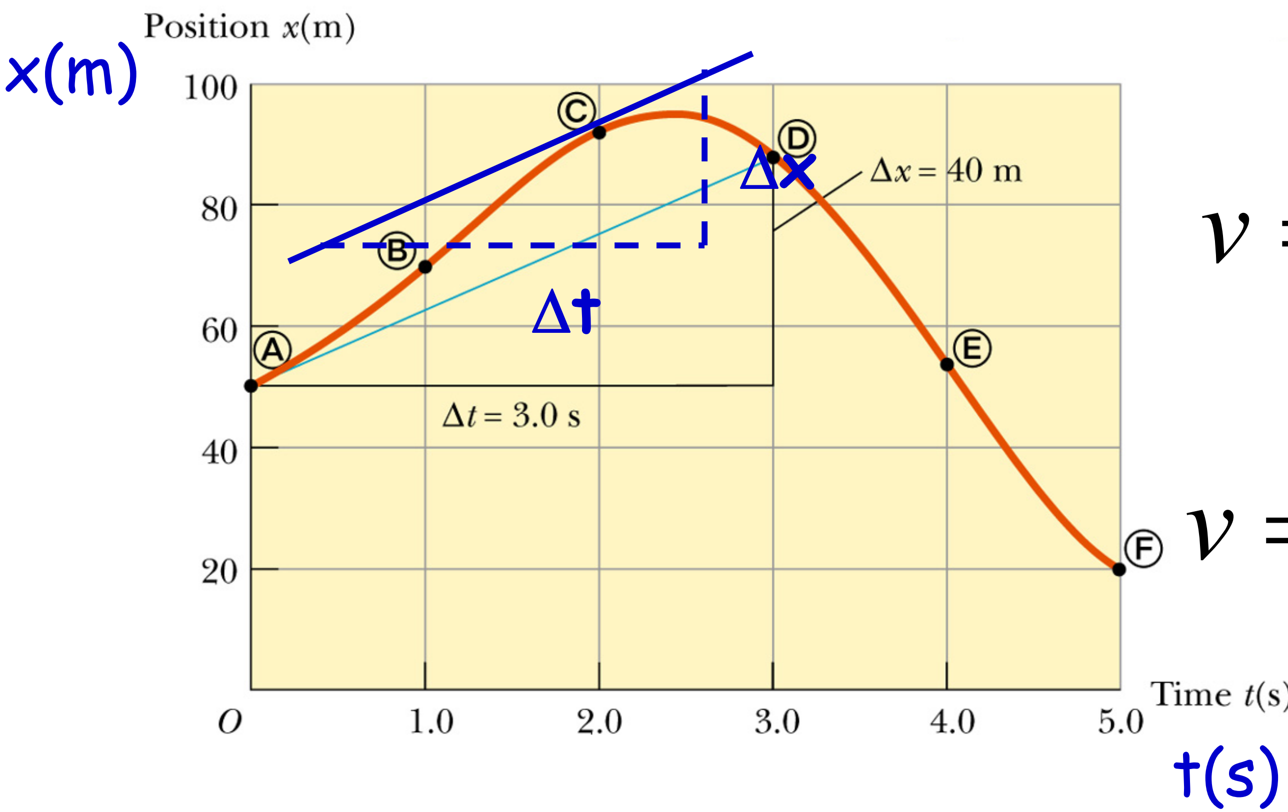
Consider the average velocity for a very small time interval with t=2.0 s in the center!

Instantaneous velocity



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous velocity



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v = \left[\frac{\Delta x}{\Delta t} \right]_{\text{tangent}}$$

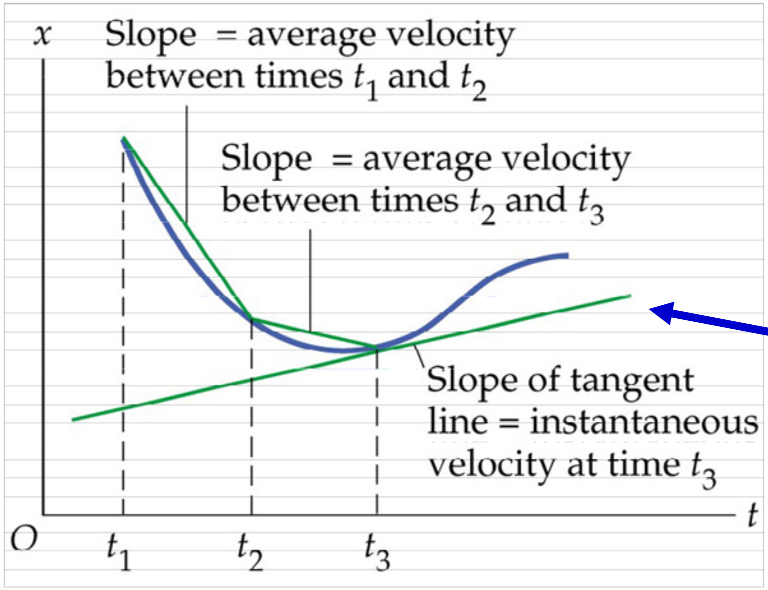
v=slope of the tangent to the x-t curve at t=2.0 s
Think about the sign!

Instantaneous Velocity

The velocity at one instant in time is known as the instantaneous velocity and is found by taking the average velocity for smaller and smaller time intervals:

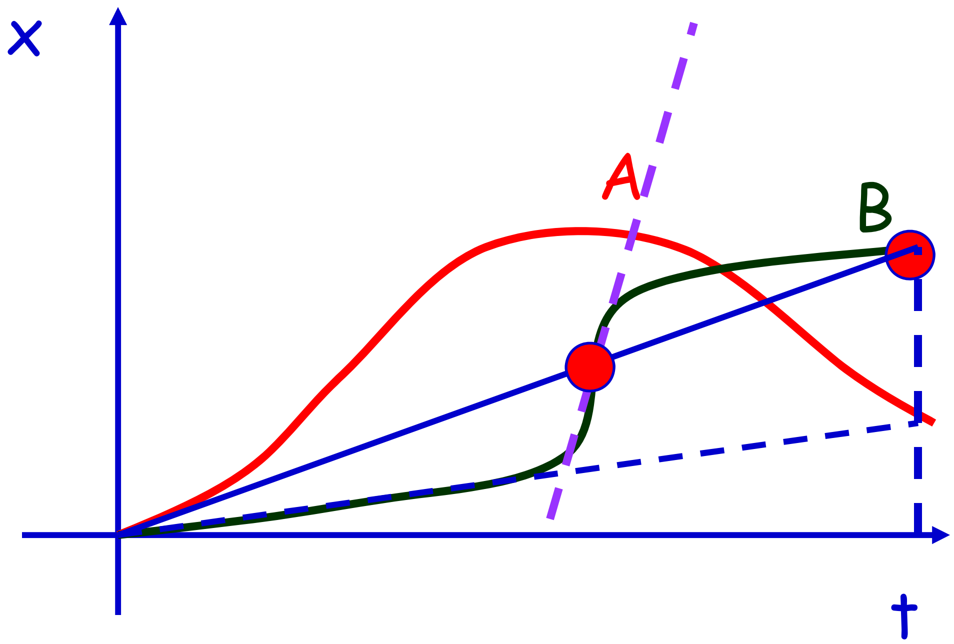
$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

The speedometer indicates instantaneous velocity ($\Delta t \approx 1$ s).



On an x vs t plot, the slope of the line tangent to the curve at a point in time is the instantaneous velocity at that time.

Question



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$v = \left[\frac{\Delta x}{\Delta t}\right]_{\text{tangent}}$$

Which object had the largest average velocity? B

Which object had the largest instantaneous velocity? B

Acceleration

Often, velocity is not constant, rather it changes with time.

The rate of change of velocity is known as acceleration.

$$a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

positive, negative or zero

This is the average acceleration.

Acceleration is a vector.

The unit of acceleration is: m/s^2

Car acceleration is often described in units miles per hour per sec

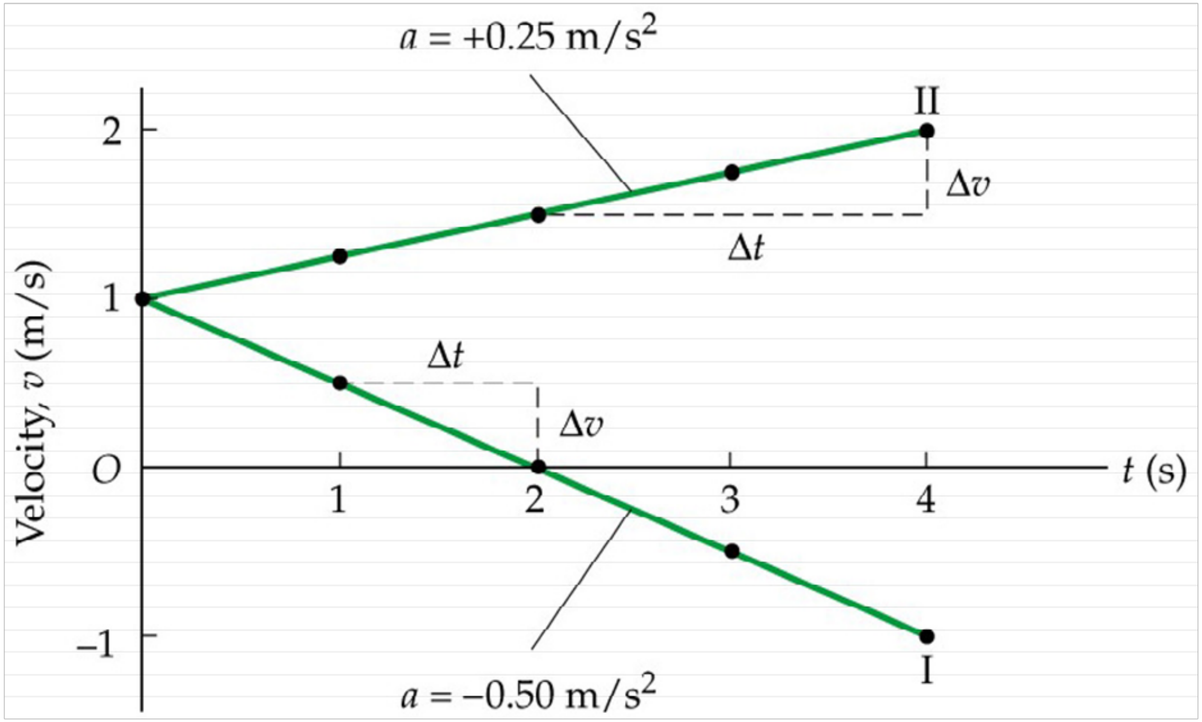
Acceleration of 0 to 60 miles per hour in 8 sec = (60mi/hr-0mi/hr)/8sec

=7.5mi/(hr*s)

$$7.5\frac{mi}{hr \cdot s} = \left[7.5\frac{mi}{hr \cdot s}\right]\left[\frac{5280\,ft}{mi}\right]\left[\frac{12in}{ft}\right]\left[\frac{0.0254m}{in}\right]\left[\frac{1hr}{3600s}\right] = 3.35\frac{m}{s^2}$$

Velocity vs. Time Plots

Graphically, acceleration can be found from the slope of a velocity vs. time curve.



For these curves, the average acceleration and the instantaneous acceleration are the same, because the acceleration is constant.

Deceleration

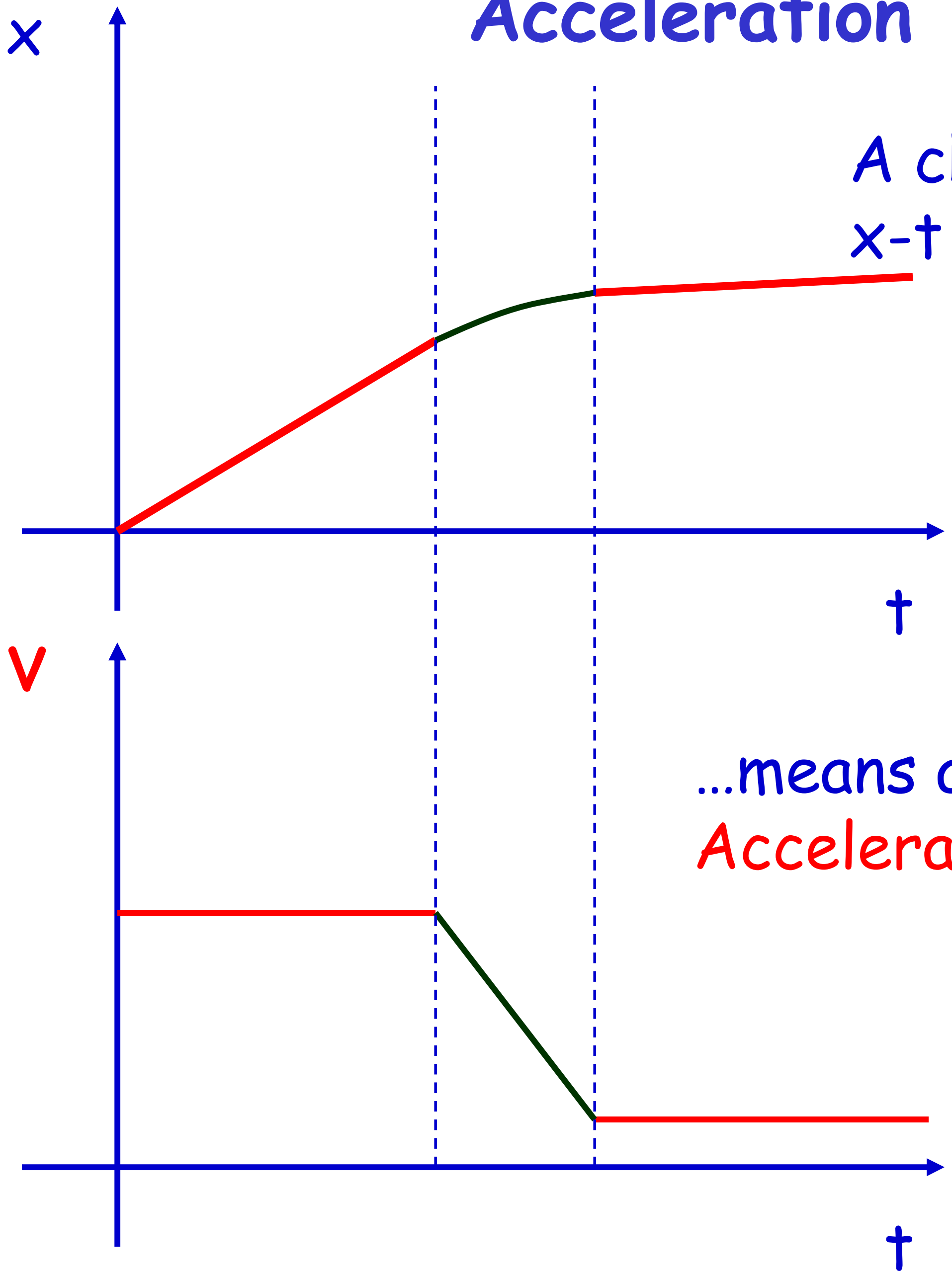
Deceleration

- refers to decreasing speed
- is not the same as negative acceleration
- occurs when velocity and acceleration have opposite signs

Example: A ball thrown up in the air. The velocity is upward but the acceleration is downward. The ball is slowing down as it moves upward. (Once the ball reaches its highest point and starts to fall again, it is no longer decelerating.)

If up is our convention for positive, then both when the ball is rising and falling, the acceleration is negative (during the instant of bounce, the acceleration is positive).

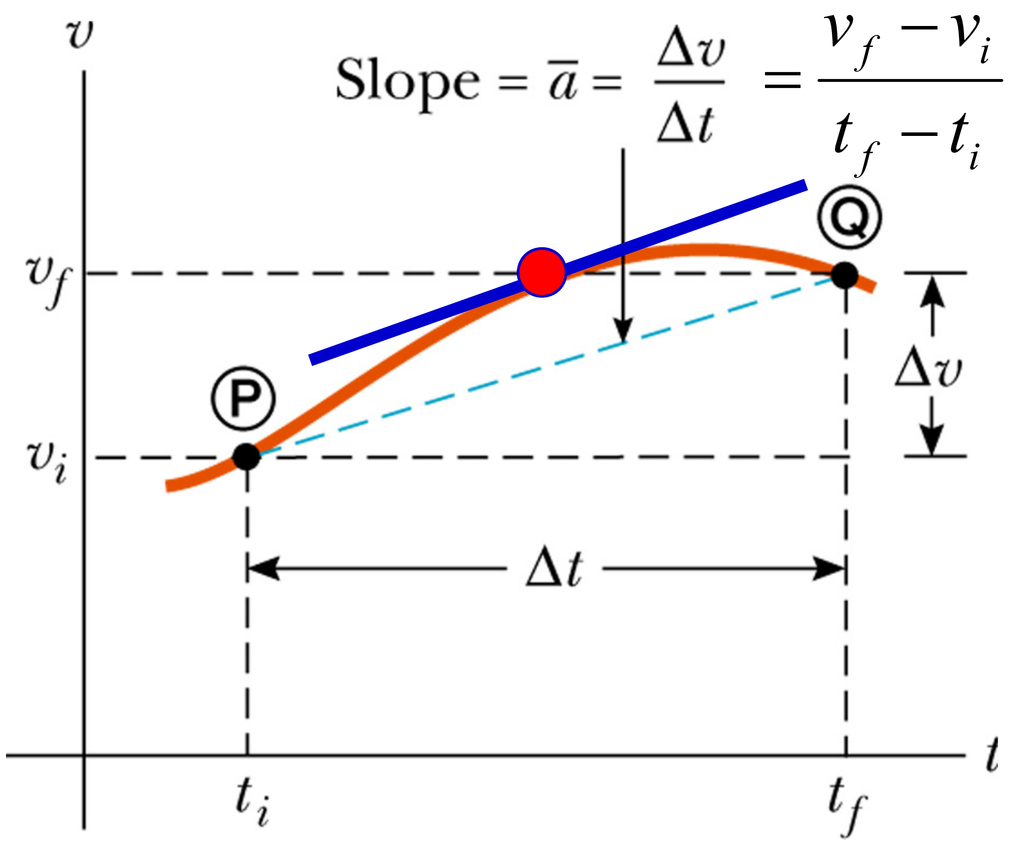
Acceleration



A change in slope in the x-t graph…

…means a change in velocity: Acceleration

Acceleration



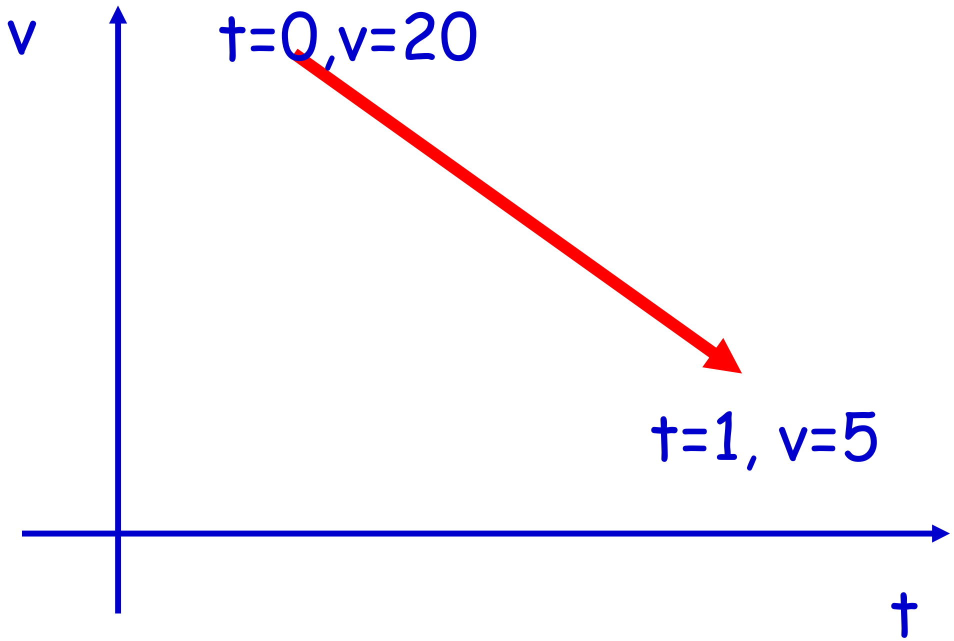
Average acceleration

Instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

$$a = \left[\frac{\Delta v}{\Delta t}\right]_{\text{tangent}}$$

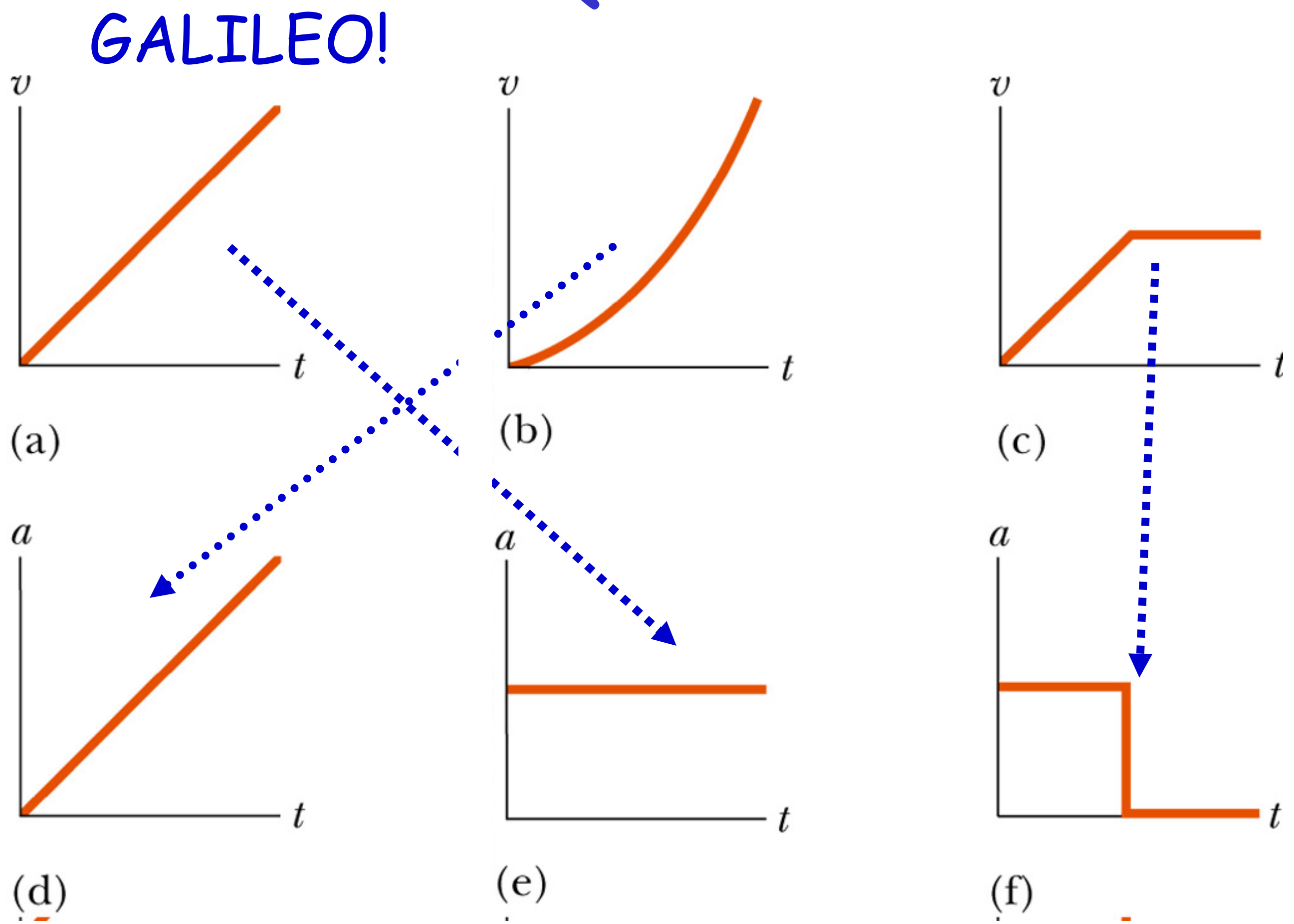
Question



What is the average acceleration between t=0 and t=1

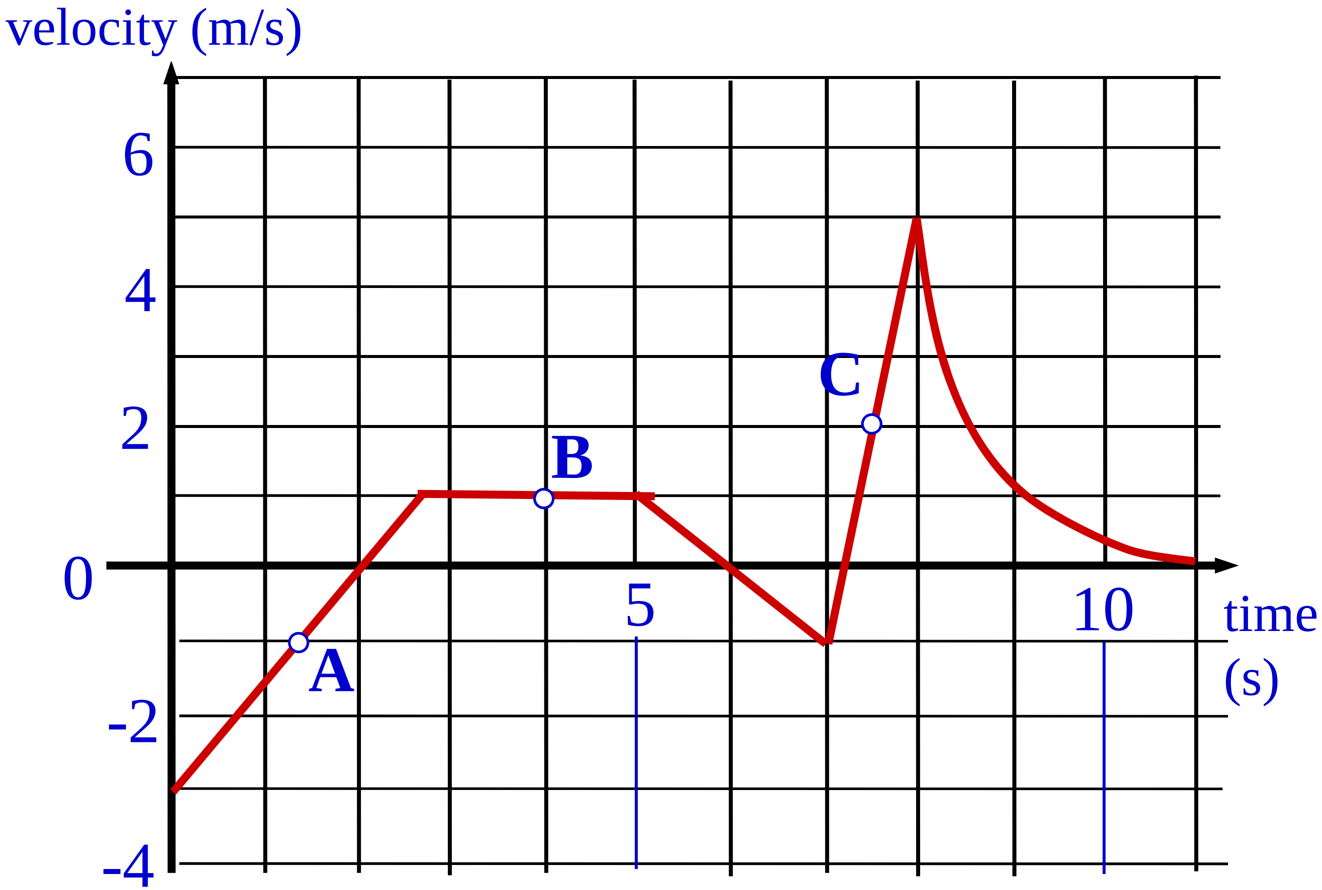
A) 15 B) -15 C) 0 D) infinity

Question



Which v-t diagram matches which a-t diagram?

Example: Velocity vs. Time Plot



1. What is the velocity at time t = 3 sec?
2. What is the velocity at point A?
3. When is the acceleration positive?
4. When is the acceleration negative?
5. When is the acceleration zero?
6. When is the acceleration constant?
7. When is there deceleration?
8. What is the acceleration at point C?
9. What is the acceleration at time t = 6 sec?
10. During what 1 s interval is the magnitude of the average acceleration greatest?

Example – be careful with signs

A car moves from a position of +4 m to a position of –1 m in 2 seconds. The initial velocity of the car is –4 m/s and the final velocity is –1 m/s.

(a) What is the displacement of the car?
(b) What is the average velocity of the car?
(c) What is the average acceleration of the car?

Answer:

(a) $\Delta x = x_f - x_i = -1\text{ m} - (+4\text{ m}) = -5\text{ m}$

(b) $v_{av} = \Delta x/\Delta t = -5\text{ m}/2\text{ s} = -2.5\text{ m/s}$

(c) $$a_{av} \equiv \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{-1\frac{\text{m}}{\text{s}} - (-4\frac{\text{m}}{\text{s}})}{2\text{ s}} = 1.5\ \text{m/s}^2$$

deceleration!

Motion with Constant Acceleration

If acceleration is constant, there are four useful formulae relating position x, velocity v, acceleration a at time t:

$$v = v_0 + at$$

$$x = x_0 + \frac{1}{2}(v_0 + v)t$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$$

(Instead of x_f, x_i, we are using x and x_0)

v_0 = initial velocity

x_0 = initial position

t_0 = initial time – assumed here to be at 0 s.

If $t_0 \neq 0$, replace t in these formulae with $t - t_0$

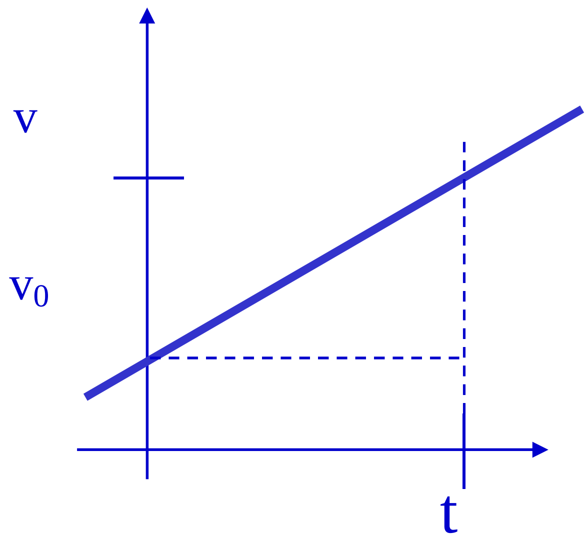
Note that we are applying restrictions and defining variables.

BE CAREFUL

WHEN USING A FORMULA!

Where do these formulae come from?

- If acceleration is constant, then a = average acceleration.
  - $a=(v-v_0) / (t-0)$
  - $(a) (t) = (v-v_0)$
  - 1): $v= v_0 + a t$
- If a = constant, then velocity *vs* time graph is a straight line
  - For a straight line graph:
  - $v_{ave} = (v+v_0)/2$
  - But $v_{ave}=(x-x_0)/(t-0)$
  - $(x-x_0)/(t-0) = (v+v_0)/2$
  - 2): $x = x_0 + (v+v_0)t/2$
  - Subsitute 1) into 2)
  - $x = x_0 + v_0 t + a t^2/2$



Example: Lets go back to our original example of the car and assume that the acceleration is constant. We found that $a = 1.5\ \text{m/s}^2$

Lets calculate the acceleration.

Recall

$$v_0 = -4\ \text{m/s},\ v = -1\ \text{m/s},\ \ t = 2\text{s}$$

$$\Rightarrow \qquad v = v_0 + at$$

$$-1\ \text{m/s} = -4\ \text{m/s} + a(2\ \text{s})$$

$$3\ \text{m/s} = a(2\ \text{s})$$

$$a = 1.5\ \text{m/s}^2$$

Freely Falling Objects

Near the earth's surface, the acceleration due to gravity g is roughly constant:

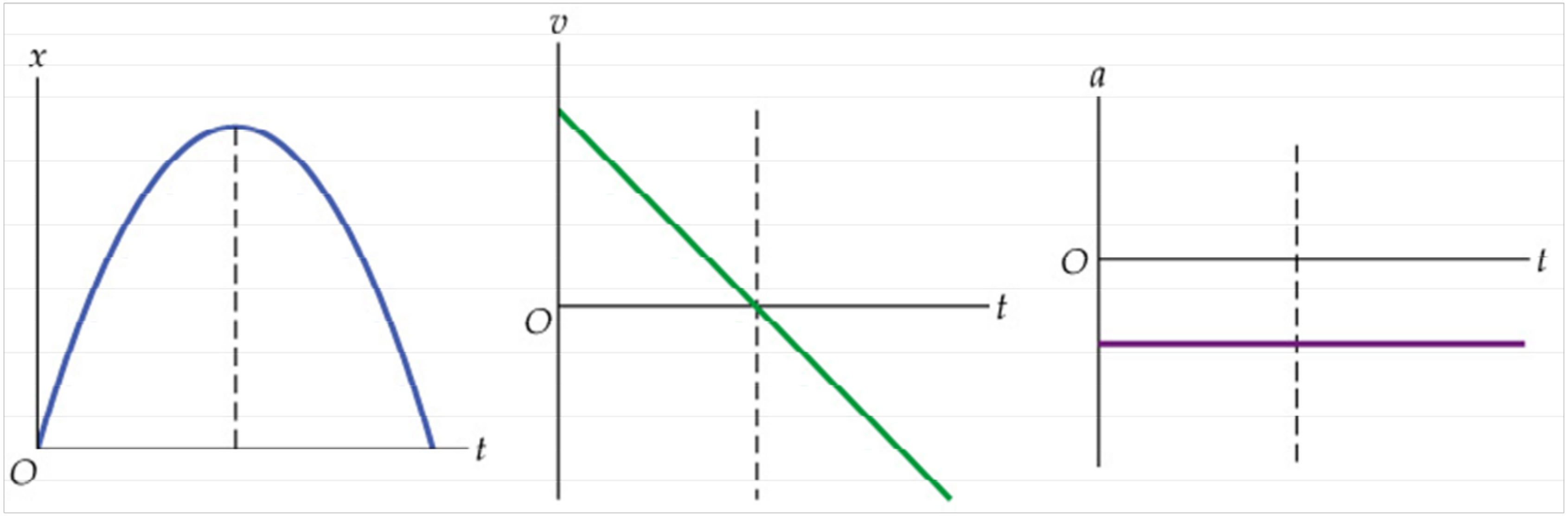
$g = a_{\text{Earth's surface}} = 9.81\ \text{m/s}^2$ toward the center of the earth

- Free fall is the motion of an object subject only to the influence of gravity (not air resistance).
- An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward

Question: What about the mass of an object?

Answer: The acceleration of gravity is the same for all objects near the surface of the Earth, regardless of mass.

Graphical example: A ball is thrown upward from the ground level.



x = ball's height above the ground

velocity is positive when the ball is moving upward

Why is acceleration negative?

Is there ever deceleration?

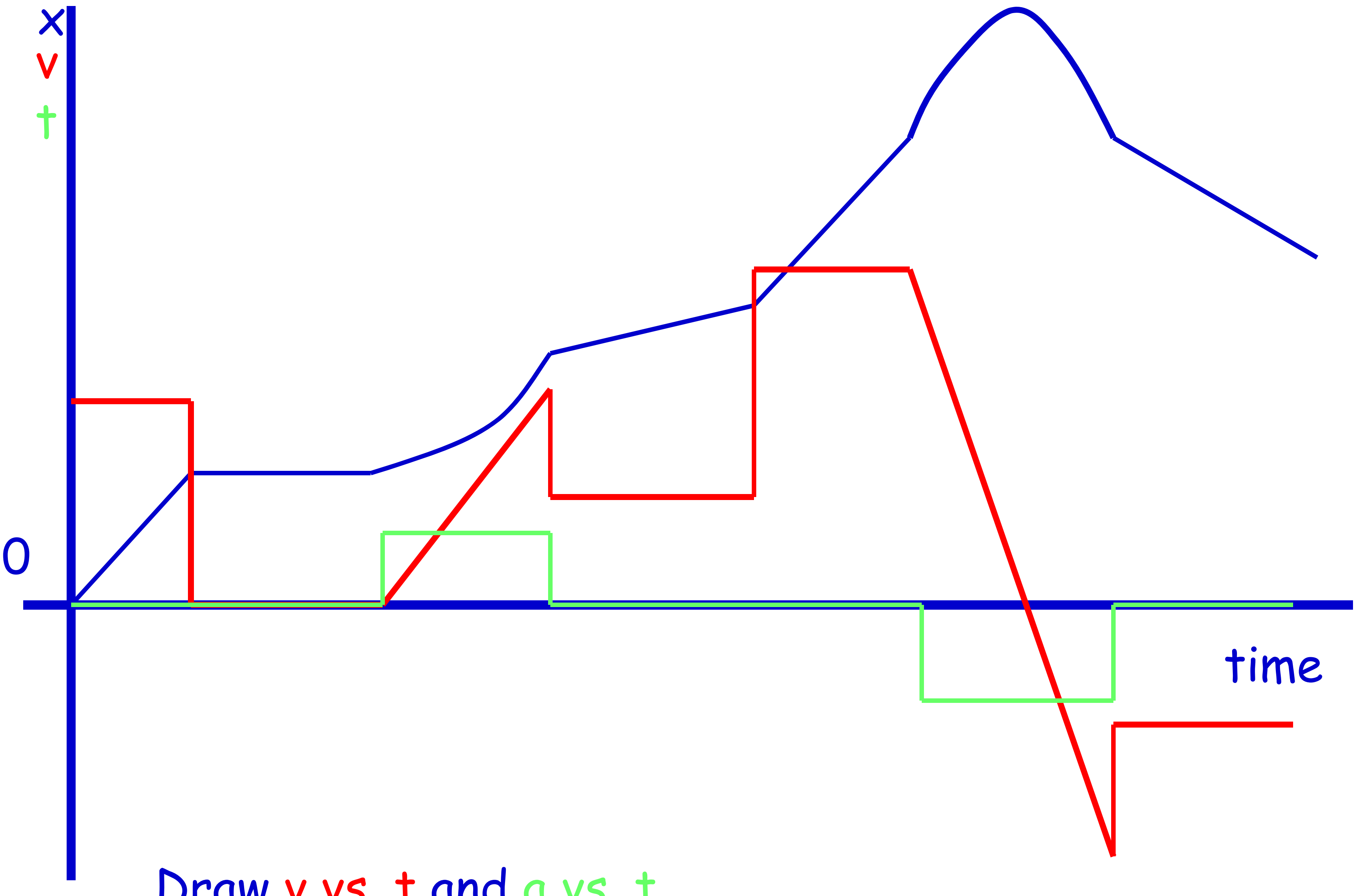
Previously…

VECTORS
- Displacement
- Average Velocity
- Instantaneous velocity
- Average acceleration
- Instantaneous acceleration

SCALARS
- Distance
- Average speed
- Instantaneous speed

MOTION DIAGRAMS

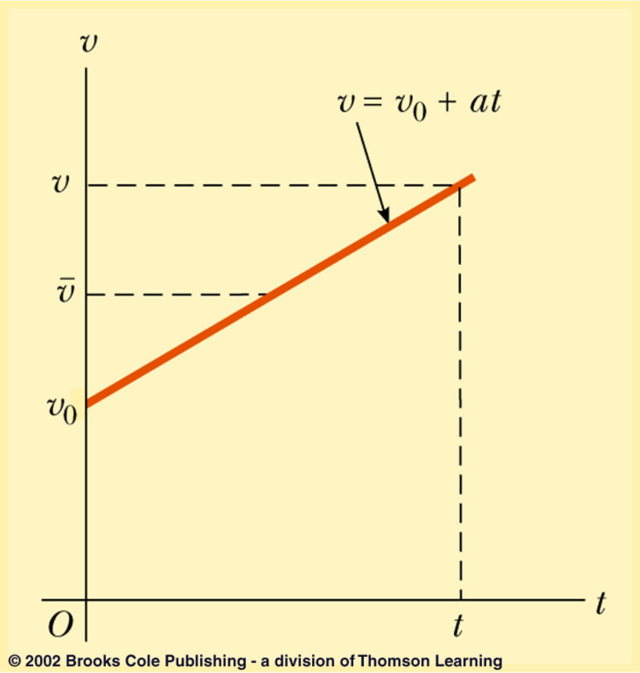
example



Draw v vs. t and a vs. t

Constant acceleration

$$v(t) = v_0 + at$$



Velocity at t=t equals...

Velocity at t=0...

Plus the gain in velocity per second
Multiplied by the time span
(every second, the velocity increases
With *a* m/s)

Constant acceleration II

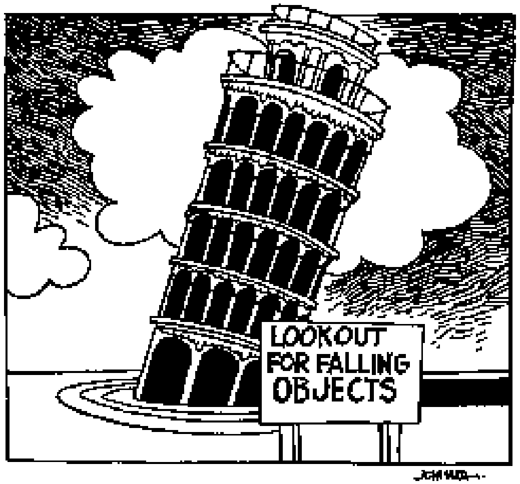
$$x(t) = x_o + \bar{v}t = x_o + \frac{(v_0 + v_t)}{2}t = x_o + v_o t + \tfrac{1}{2}at^2$$

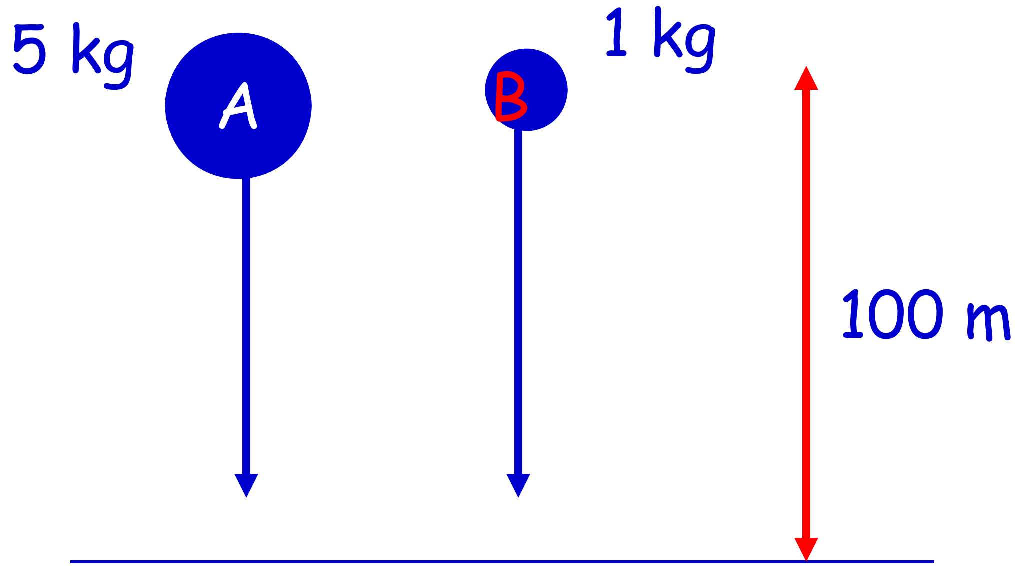
Start position plus average speed multiplied by time

Substitute $\bar{v} = \dfrac{v_0 + v_t}{2}$

Substitute $v(t) = v_0 + at$

Free fall



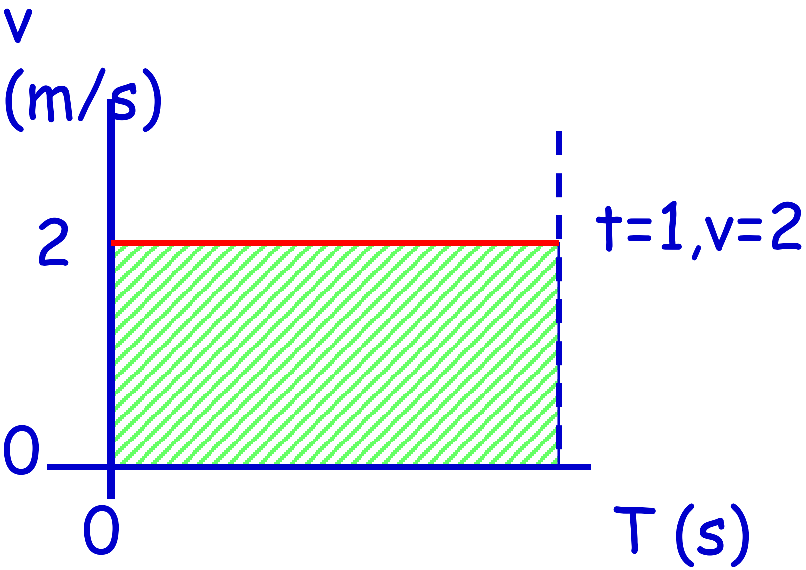


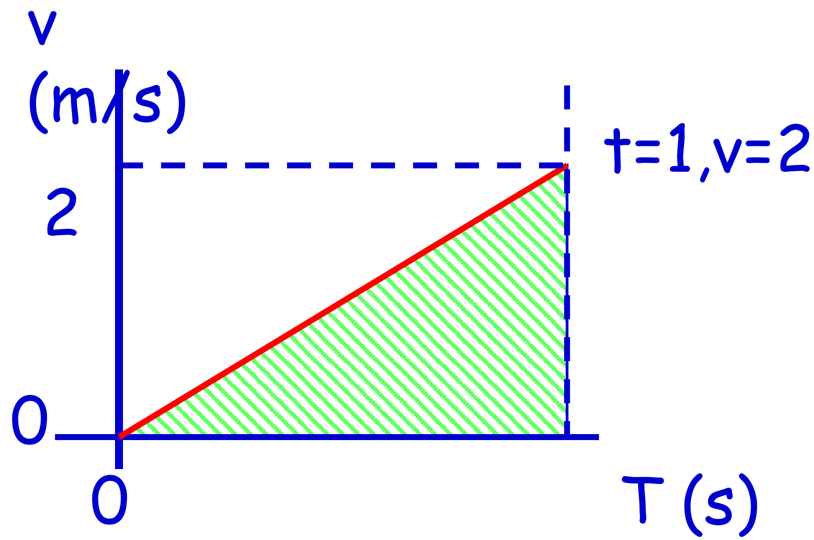
$$x(t) = x_o + v_o t + \tfrac{1}{2} a t^2$$

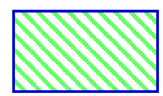
$$v(t) = v_0 + at$$

a is the the acceleration felt due to gravitation (commonly called $g=9.8$ m/s^2)

Why no mass dependence???



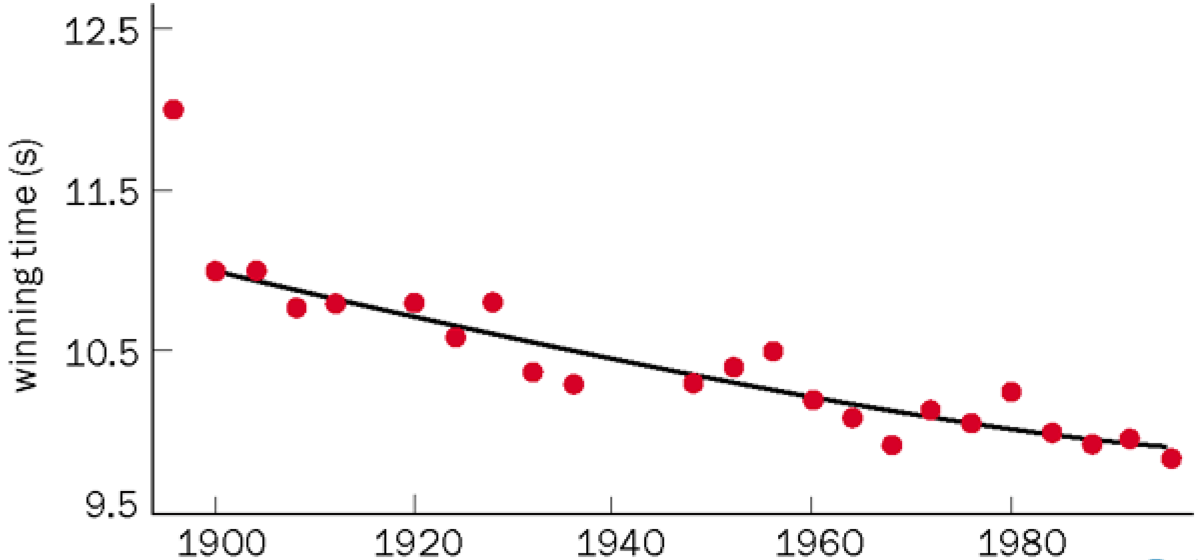


1)What is the distance covered in 1 second?
2)What is the area indicated by  ?

Q	1.	2.
a)	1.	1.
b)	1.	2.
c)	2.	1.
d)	2.	2.

The area under the v-t curve is equal to the displacement of the object!

Kinematics in sports



100 m dash: what is the best strategy?

After long training Ben Lewis can accelerate with $a=3.00\ m/s^2$ over a distance of 20.0 m. Over the remaining 80.0 m, he can maintain this top-speed.

A) After how many seconds reaches Ben top-speed?

B) What is his speed at that time?

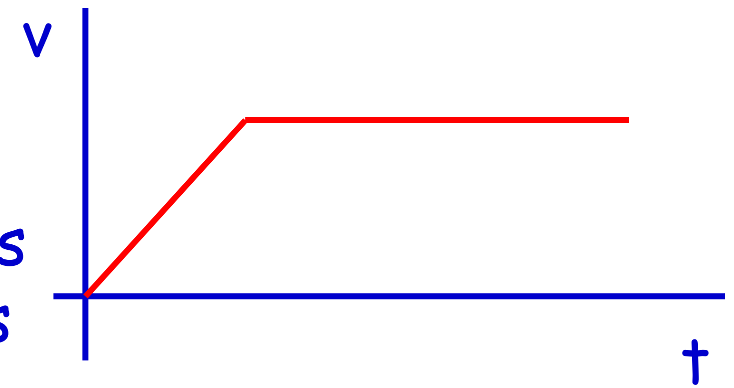
C) In how much time does he cross the finish line?

A) $20.0\ m = 1/2*3.00*t^2$ $\quad$ $t=3.65\ s$

B) $v(t=3.65)=3.00*t=10.95\ m/s$

C) last 80 m: $t=80/10.95=7.30\ s$

total time: $3.65+7.30=10.95\ s$



After a lot of training...

Ben manages to accelerate the first 3.65 s with $a=4.00\ m/s^2$. After reaching his top-speed, he cannot maintain it however, and slowly de-accelerates ($a=-0.4\ m/s^2$). Did his total time improve over 100 m?

A) What distance does Ben cover while accelerating?

B) What is his speed at that time?

C) How long does is take to cover to remaining distance and what is his total time?

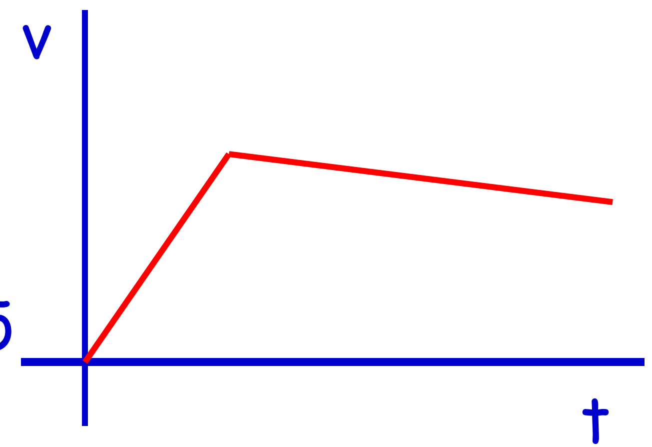
A) $x=1/2*4.00*3.65^2=26.6$ m

B) $v(t=3.65\ s)=4.00*3.65=14.6$ m/s

C) $100=26.6+14.6*t-1/2*0.4*t^2$

$t^2-73*t+372=0$, so $t=5.51$ or $t=67.5$

total time: $3.65+5.51=9.16$



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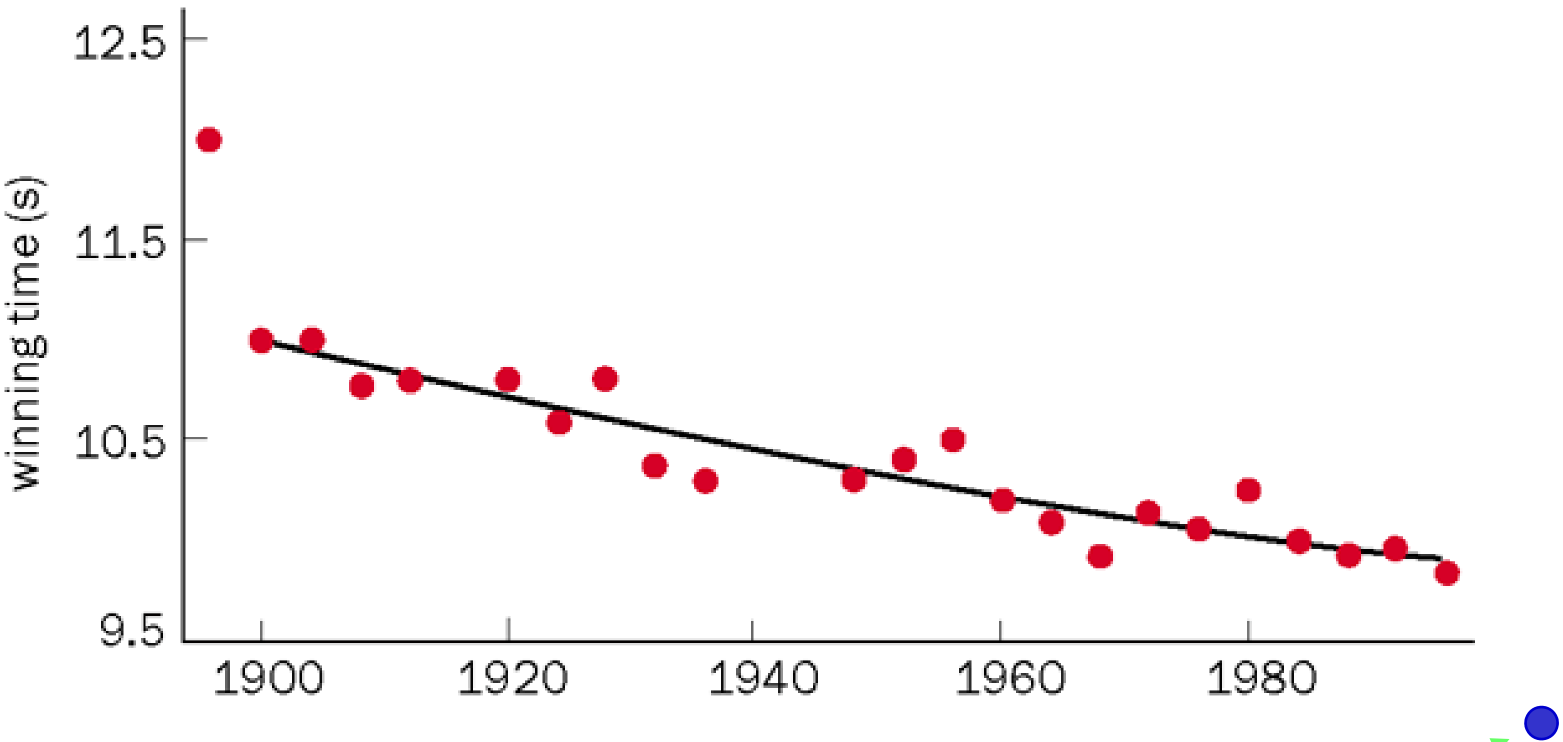

Ben Lewis!

Problem Solving Strategy

- Make a list of given quantities
- Make a sketch
- Draw coordinate axes – identify the positive direction
- Identify what is to be determined
- Be consistent with units
- Check that the answer seems reasonable
- **Don't panic.**