Simple harmonic motion

displacement x

5cm

-5cm

2   4   6   8   10   time (s)
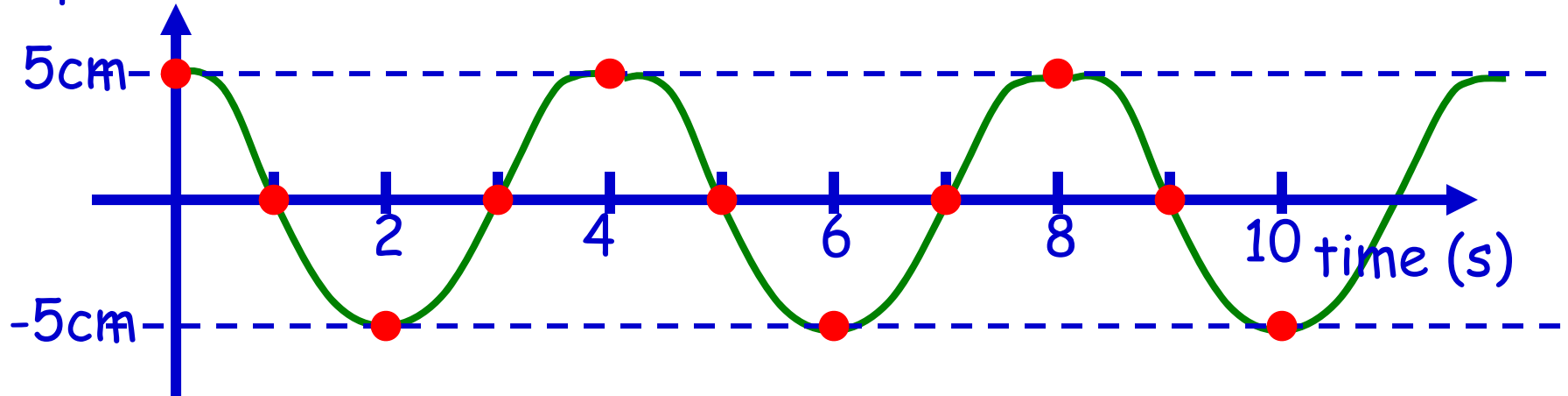
a) what is the amplitude of the harmonic oscillation?
b) what is the period of the harmonic oscillation?
c) what is the frequency of the harmonic oscillation?

a) Amplitude: 5 cm (0.05 m)
b) period: time to complete one full oscillation: 4s
c) frequency: number of oscillations per second=1/T=0.25 s

1

example

A mass of 1 kg is hung from a spring. The spring stretches by 0.5 m. Next, the spring is placed horizontally and fixed on one side to the wall. The same mass is attached and the spring stretched by 0.2 m and then released. What is the acceleration upon release?

1st step: find the spring constant k
$F_{spring} = -F_{gravity}$ or $-kd = -mg$
$k = mg/d = 1*9.8/0.5 = 19.6$ N/m

2nd step: find the acceleration upon release
Newton's second law: $F=ma \Rightarrow -kx=ma \Rightarrow a=-kx/m$
$a=-19.6*0.2/1=-3.92$ m/s^2

An example

A 0.4 kg object, connected to a light spring with a spring constant of 19.6 N/m oscillates on a frictionless horizontal surface. If the spring is compressed by 0.04 and then released determine: a) the maximum speed of the object b) the speed of the object when the spring is compressed by 0.015 m c) when it is stretched by 0.015 m d) for what value of x does the speed equal one half of the maximum speed?

a) $v = \sqrt{[(A^2-x^2)k/m]}$ (speed is always positive!)
   maximum if $x=0$: $\sqrt{[A^2k/m]}=0.04\sqrt{(19.6/0.4)}=0.28$ m/s

b) $v=\sqrt{[(A^2-x^2)k/m]}$ at $x=-0.015$
   $v=\sqrt{[((0.04)^2-(-0.015)^2)19.6/0.4]}=0.26$ m/s

c) same as b)

d) $\sqrt{[(A^2-x^2)k/m]}=0.28/2=0.14$ $x=\sqrt{(A^2-0.14^2m/k)}=0.035$m

Example

A mass of 0.2 kg is attached to a spring with k=100 N/m. The spring is stretched over 0.1 m and released.

a) What is the angular frequency (ω) of the corresponding circular motion?

b) What is the period (T) of the harmonic motion?

c) What is the frequency (f)?

d) What are the functions for x,v and t of the mass as a function of time? Make a sketch of these.

a) $\omega=\sqrt{(k/m)}=$ $\omega=\sqrt{(100/0.2)}=22.4$ rad/s
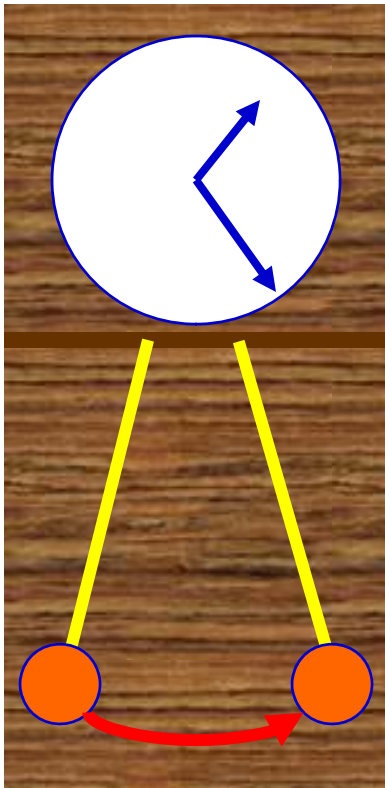
b) $\omega=2\pi/T$ T= $2\pi/\omega=0.28$ s

c) $\omega=2\pi f$ f=$\omega/2\pi=3.55$ Hz (=1/T)

d) $x_{harmonic}(t)=A\cos(\omega t)=0.1\cos(0.28t)$

$v_{harmonic}(t)=-\omega A\sin(\omega t)=-0.028\sin(0.28t)$

$a_{harmonic}(t)=-\omega^2 A\cos(\omega t)=-0.0078\cos(0.28t)$
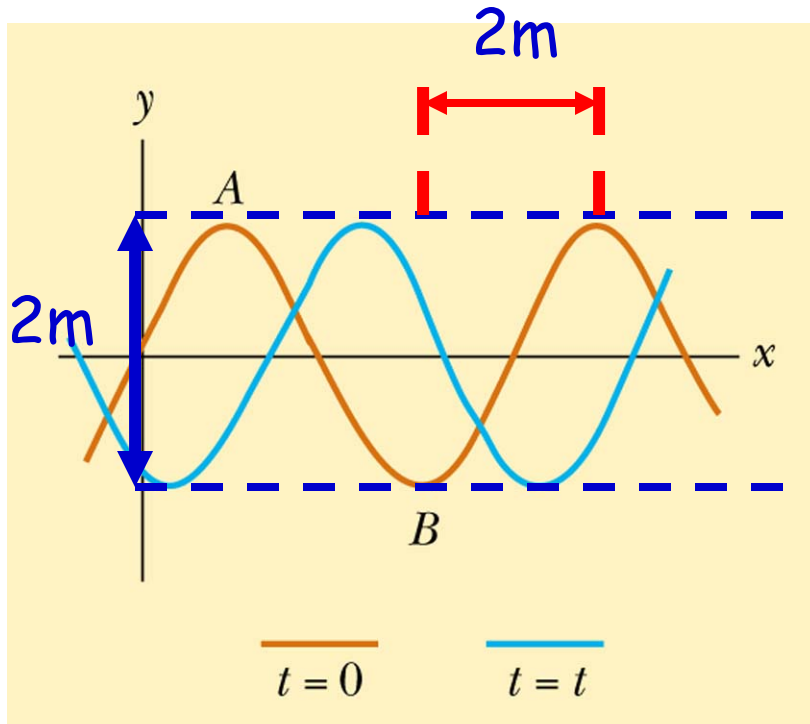
4

Example: a pendulum clock

The machinery in a pendulum clock is kept in motion by the swinging pendulum.
Does the clock run faster, at the same speed, or slower if:
a) The mass is hung higher
b) The mass is replaced by a heavier mass
c) The clock is brought to the moon
d) The clock is put in an upward accelerating elevator?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

	L↓	m↑	moon	elevator
faster	●			●
same		●		
slower			●	

Example

2m

2m

A traveling wave is seen to have a horizontal distance of 2 m between a maximum and the nearest minimum and vertical height of 2 m. If it moves with 1 m/s, what is its:
a) amplitude
b) period
c) frequency

a) amplitude: difference between maximum (or minimum) and the equilibrium position in the vertical direction (transversal!) A=2 m/2=1 m

b) v=1 m/s, λ=2*2 m=4 m   T=λ/v=4/1=4 s

c) f=1/T=0.25 Hz