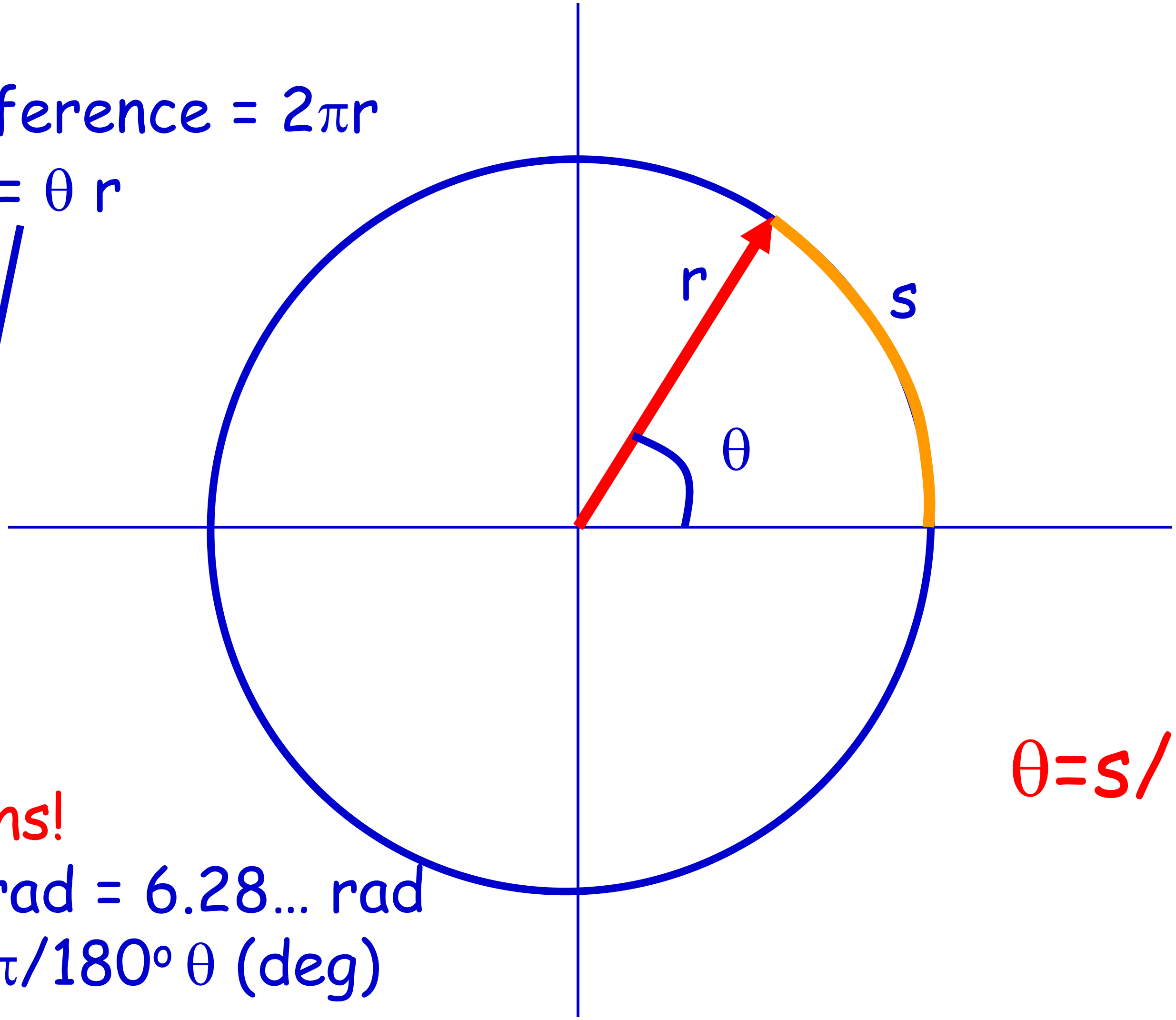
Radians & Radius

Circumference = $2\pi r$

Part $s = \theta\, r$



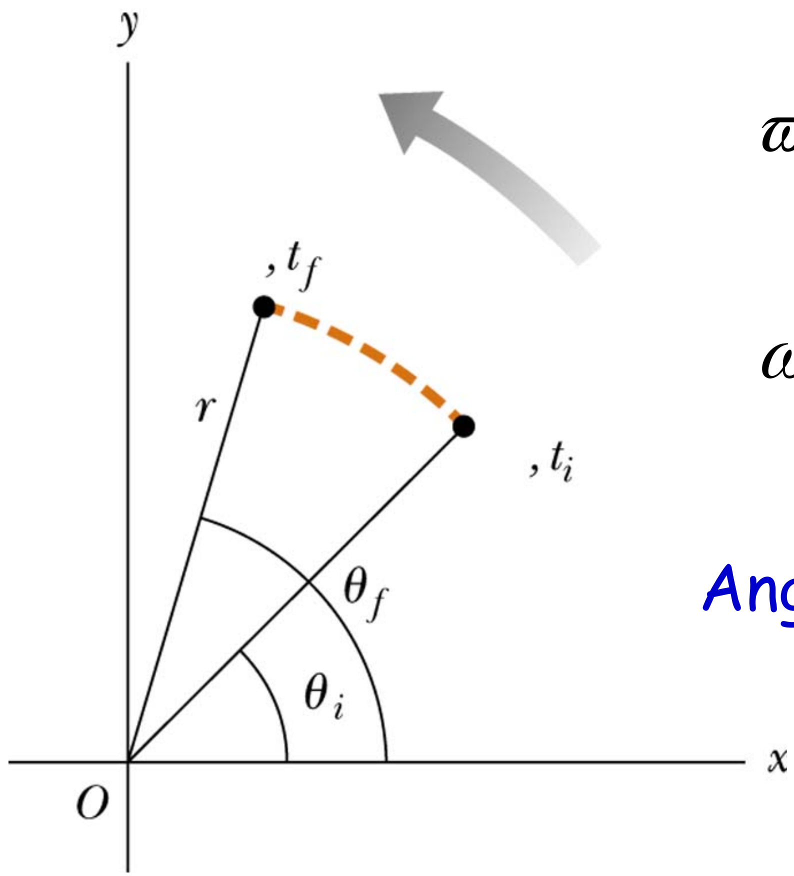
$\theta = s/r$

θ in radians!

$360^{\circ} = 2\pi$ rad = 6.28… rad

$\theta\,(\text{rad}) = \pi/180^{\circ}\;\theta\,(\text{deg})$

Angular speed and acceleration



$$\varpi = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Average angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular velocity

Angular velocity : rad/s
rev/s
rpm (revolutions per minute)

example

What is the angular velocity of earth around the sun? Give in rad/s, rev/s and rpm

Answer: in 1 year, the earth makes one full orbit around the sun.

$$\varpi = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

rad/s	rev/s	rpm
= 2π rad/1 year	1 rev/1 year	1 rev/1 year
= 2π rad/(3.2E+7 s)	1 rev/(3.2E+7 s)	1 rev/(5.3E+5 min)
= 2.0E-07 rad/s	3.1E-8 rev/s	1.9E-6 rpm

Angular acceleration

Definition: The change in angular velocity per time unit

$$\overline{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Average angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular acceleration

Unit: rad/s^2

Equations of motion

Linear motion

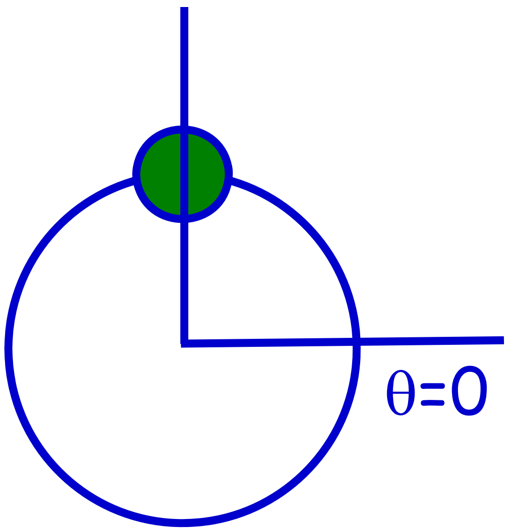
$X(t)=x(0)+v(0)t+\frac{1}{2}at^2$

$V(t)=V(0)+at$

Angular motion

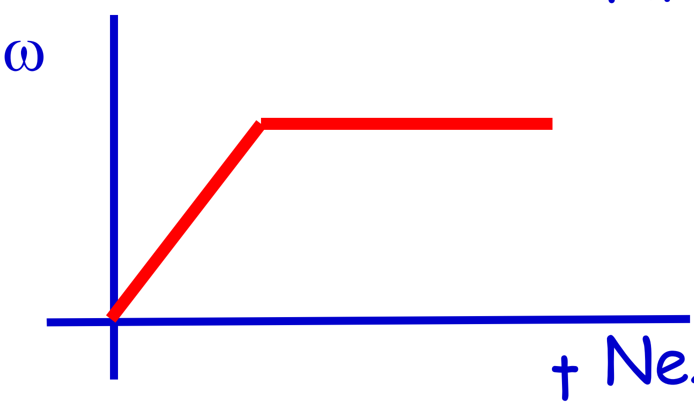
$\theta(t)= \theta(0)+\omega(0)+t\frac{1}{2}\alpha t^2$

$\omega(t)= \omega(0)+\alpha t$



example

A person is rotating a wheel. The handle is initially at $\theta=90^o$. For 5s the wheel gets an constant angular acceleration of 1 rad/s^2. After that the angular velocity is constant. Through what angle will the wheel have rotated after 10s.



First 5s: $\theta(5)= \theta(0)+\omega(0)t+\frac{1}{2}\alpha t^2$

$= \pi/2+\ \ 0\ \ + \frac{1}{2}1(5)^2$

$= \pi/2+12.5=14.1$ rad $(=806^o)$

$\omega(5)= \omega(0)+\alpha t$

$= 1t= 5$ rad/s

Next 5s: $\theta(5)= 14.1+5t$

$=$ 39.1 rad $(=2240^0=6.2$ rev$)$

Angular ⟷ Linear velocities

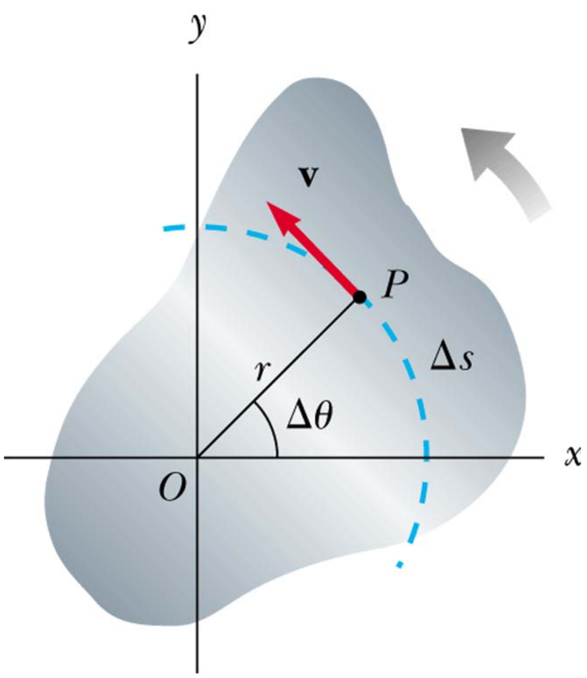
$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\boxed{\frac{\Delta s}{\Delta t}}$$

V

$$\omega = \frac{v}{r}$$

$$v = \omega r$$



V is called the tangential velocity

Angular $\longleftrightarrow$ linear acceleration

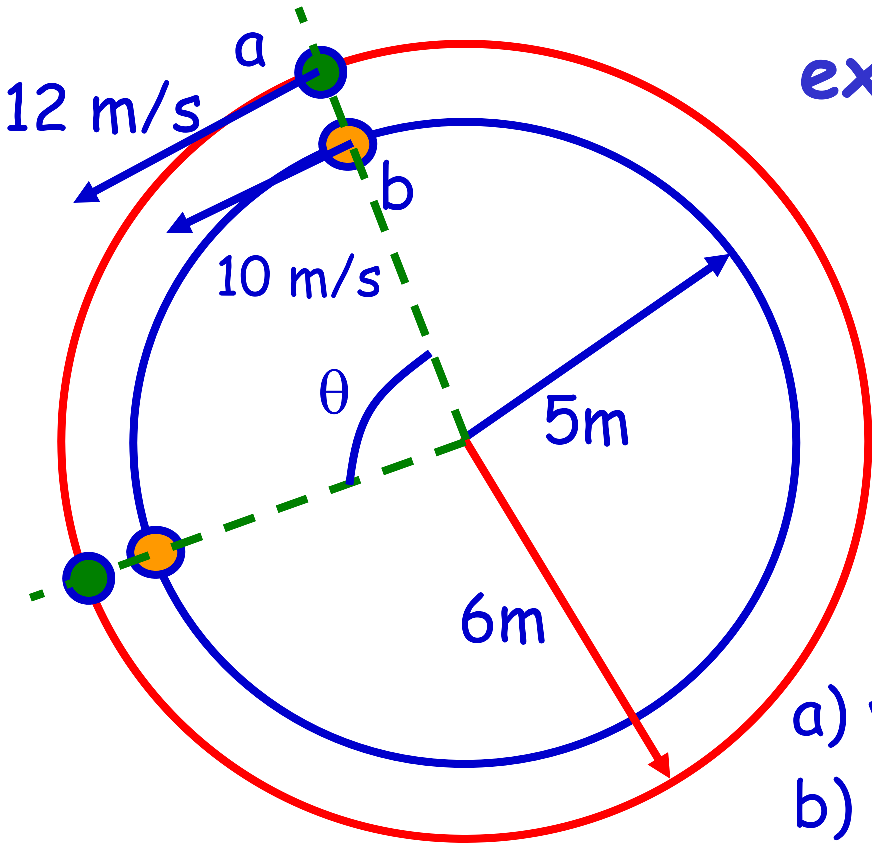
$$v = \omega r$$ velocity

$$\Delta v = \Delta\omega \cdot r$$ Change in velocity

$$\frac{\Delta v}{\Delta t} = \frac{\Delta\omega}{\Delta t} r$$ Change in velocity per time unit

$$\boxed{a = \alpha r}$$ acceleration

The linear acceleration equals the angular acceleration times the radius of the orbit



example

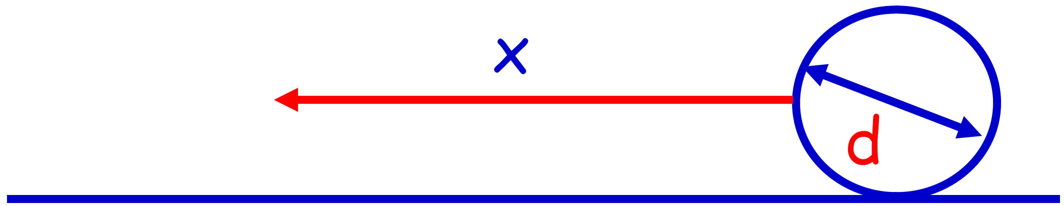
The angular velocity of a is 2 rad/s.
a) What is its tangential velocity?
If b is keeping pace with a,
b) What is its angular velocity?
c) What is its linear velocity?

a) $v = \omega r = 2*6 = 12$ m/s

b) Must be the same: 2 rad/s

c) $v = \omega r = 2*5 = 10$ m/s

A rolling coin



ω_0=18 rad/s
α=-1.80 rad/s
d=0.02 m

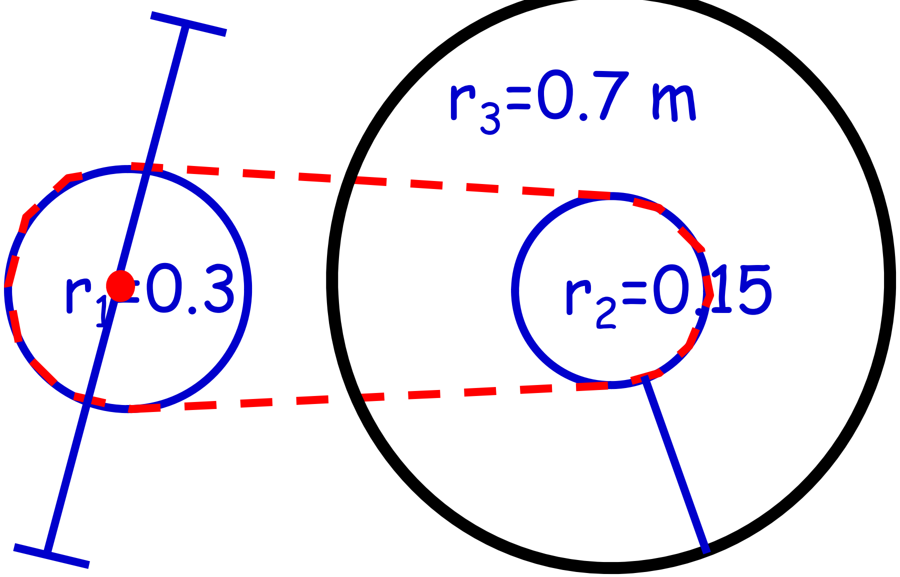
a) For how long does the coin roll?
b) What is the average angular velocity?
c) How far does the coin roll before coming to rest?

a) $\omega(t) = \omega(0) + \alpha t$ 0=18-1.8t t=10 s

b) $\overline{\omega} = (\omega(0) + \omega(10))/2$=18/2=9 rad/s

c) $\overline{v} = \overline{\omega} r$ = 9*0.01=0.09 m/s x=vt=0.09*10=0.9 m

gears



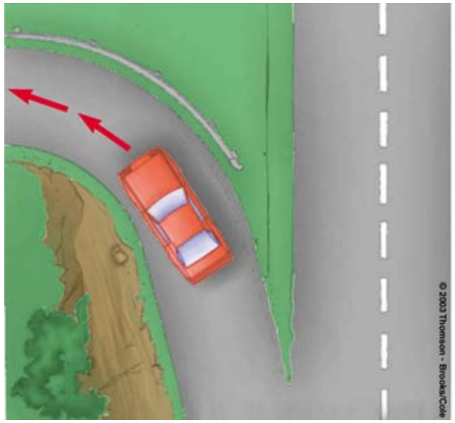
If ω_1=3 rad/s, how fast is the bike going?
b) What if r_1=0.1 m ?

$V_1 = \omega_1 r_1 = 3*0.3 = 0.9$ m/s
$V_1 = V_2$ (because of the chain)
$\omega_2 = V_2/r_2 = 0.9/0.15 = 6$ rad/s
$\omega_3 = \omega_2$ (connected)
$V_3 = \omega_3 r_3 = 6*0.7 = 4.2$ m/s

b)

$V_1 = \omega_1 r_1 = 3*0.1 = 0.3$ m/s
$V_1 = V_2$ (because of the chain)
$\omega_2 = V_2/r_2 = 0.3/0.15 = 2$ rad/s
$\omega_3 = \omega_2$ (connected)
$V_3 = \omega_3 r_3 = 2*0.7 = 1.4$ m/s

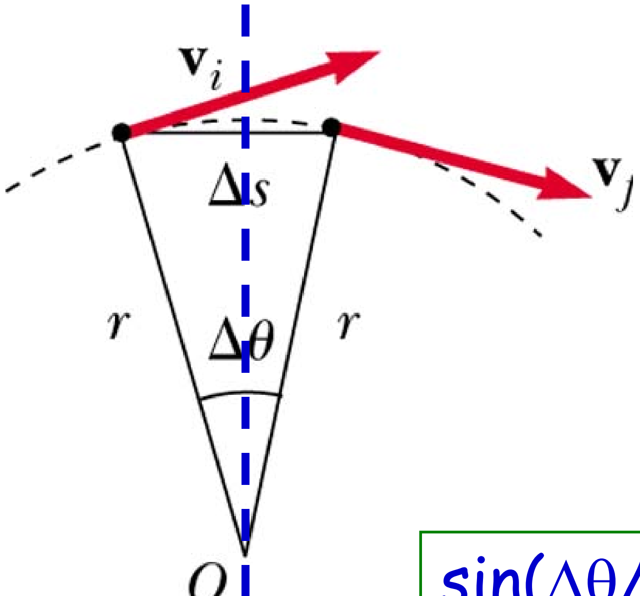
Driving a car through a bend



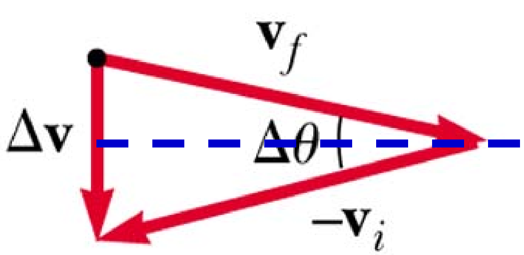
Is there a force that pushes you away from the center of the circle?

- Newton's first law: If no net force is acting on an object, it will continue with the same velocity (inertia of mass)
- Velocity is a vector (points to a direction)
- If no net force is acting on an object, it will not change its direction.
- A force is acting on the car (steering+friction) but you tend to go in the same direction as you were going!
- It is not a force that pushes you, but the lack of it!
- The side door will keep you from falling out: it exerts a force on you and you exert a force on the door ($F_{21}=-F_{12}$)

Centripetal acceleration



(a)



(b)

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

The change in velocity is not the change in speed but in direction.

$\sin(\Delta\theta/2) \sim (\Delta s/2)/r$

$\sin(\Delta\theta/2) = (\Delta v/2)/v$

$(\Delta s/2)/r = (\Delta v/2)/v$

$\frac{\Delta v}{\Delta t} = \frac{\Delta s}{\Delta t} * (v/r)$

$v \approx \Delta s/\Delta t$

$a_c = v^2/r$

Centripetal acceleration

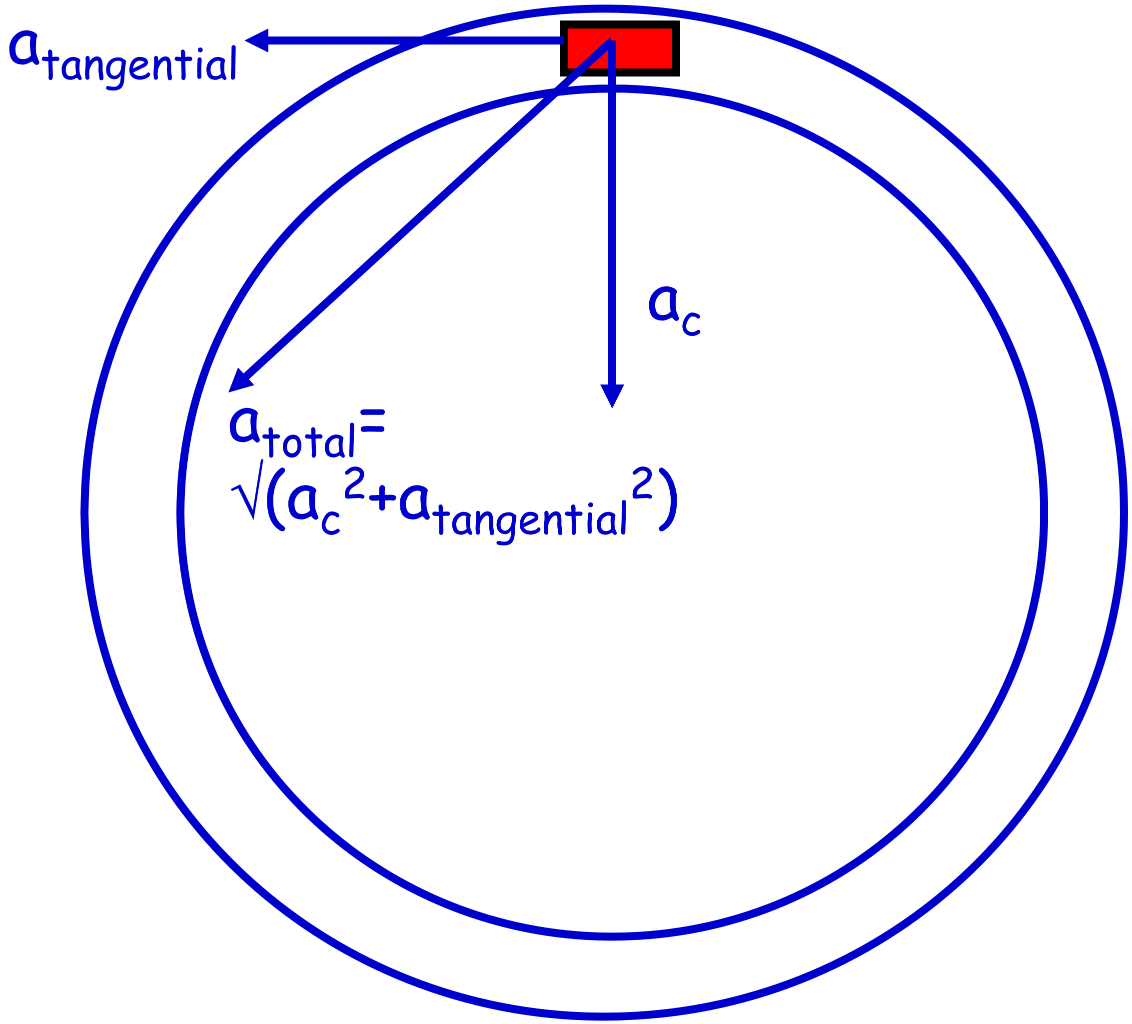
$\boxed{a_c=v^2/r}$ directed to the center of the circular motion

Also $v=\omega r$, so $\boxed{a_c=\omega^2 r}$

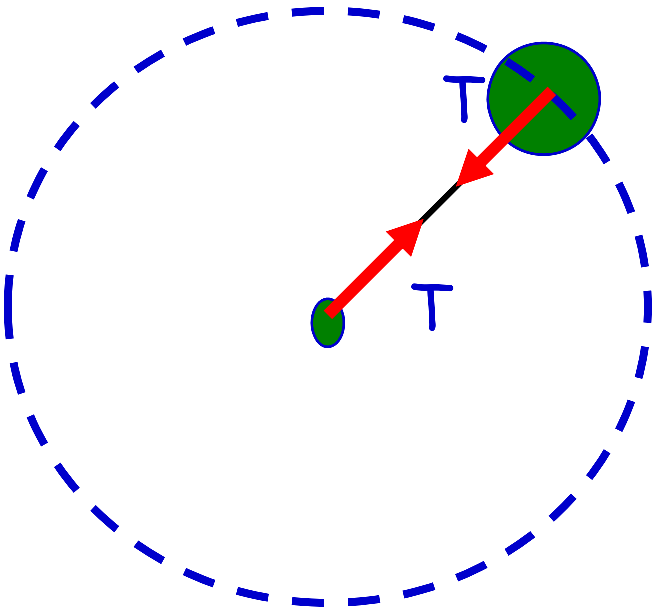
This acceleration can be caused by various forces:
- gravity (objects attracted by earth)
- tension (object making circular motion on a rope)
- friction (car driving through a curve)
- etc

This acceleration is NOT caused by a mysterious force

A race car accelerating on a track.



Forces that can cause centripetal acceleration.



Object swinging on a rope.

$\Sigma F=ma$

$T=ma_c$

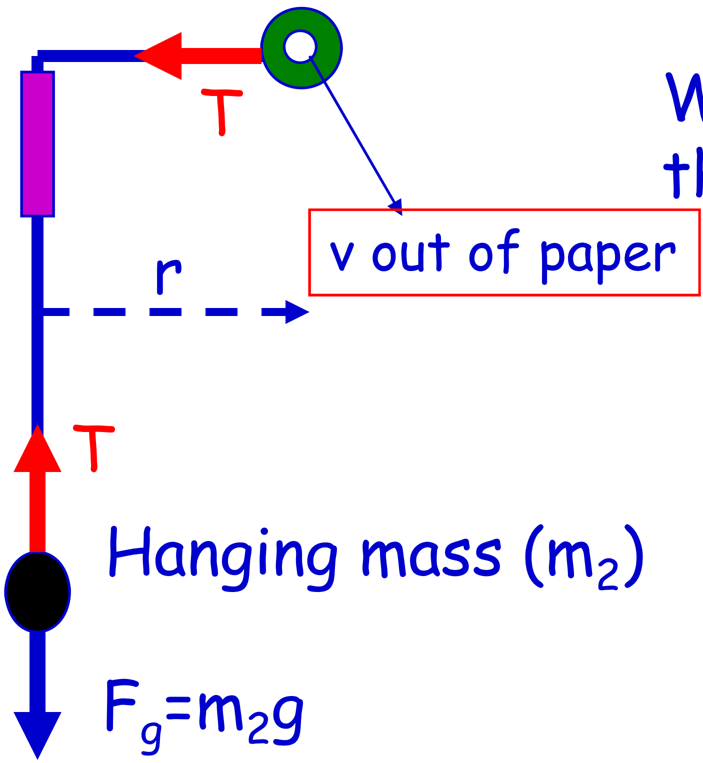
$T=mv^2/r=m\omega^2 r$

An object with m=1 kg is swung with a rope of length 3 m around with angular velocity ω=2 rad/s. What is the tension in the rope?

$T=m\omega^2 r=1*2^2*3=12$ N

Lifting by swinging

Swinging mass (m_1) with velocity v



What is the relation between v and r that will keep m_2 stationary?

m_1: $T=m_1a$

m_2: $T=m_2g$

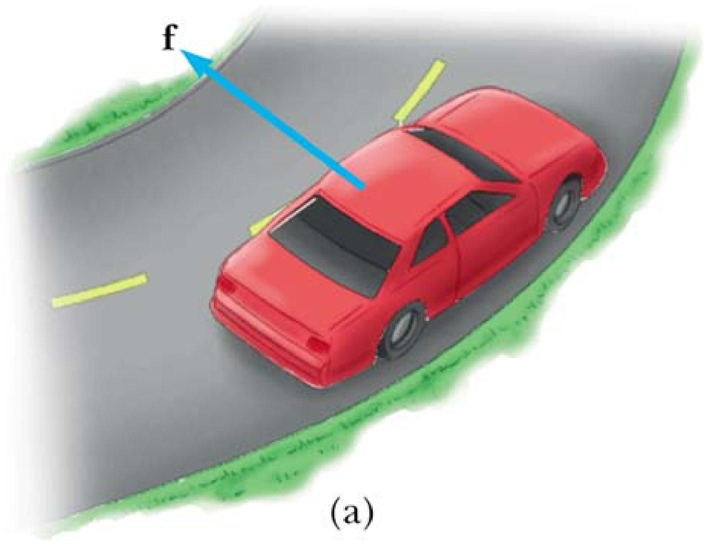
$a=m_2g/m_1$

Also: $a_c=v^2/r$

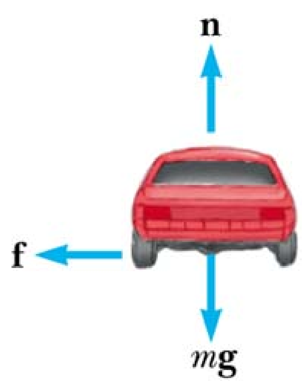
$v^2/r= m_2g/m_1$

If m_1 slows down, r must go down so m_2 sinks.

A car going through a bend



(a)

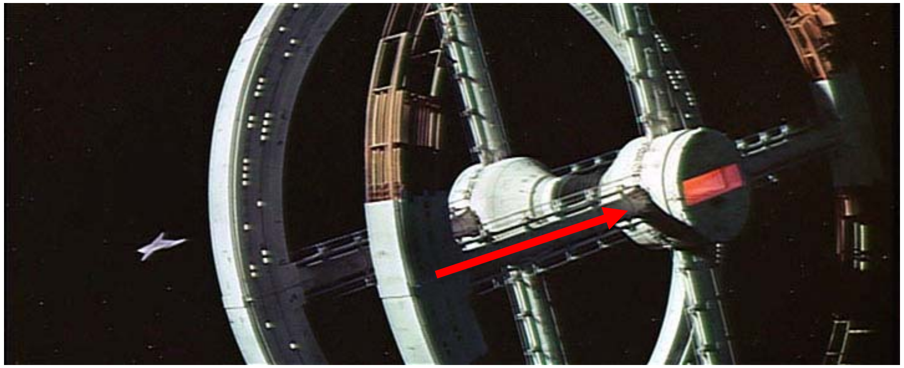


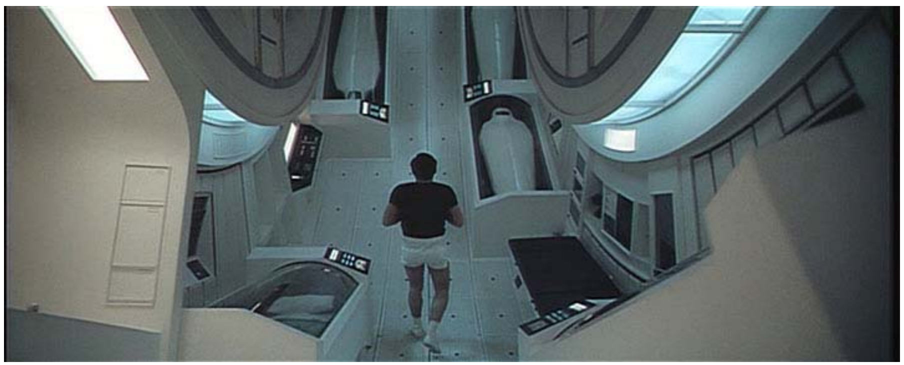
(b)

A car is passing through a bend with radius 100 m. The kinetic coefficient of friction of the tires on the road is 0.5. What is the maximum velocity the car can have without flying out of the bend?

$\Sigma F = ma$
$\mu_k n = m a_c$
$\mu_k mg = mv^2/r$
$0.5 * 9.81 = v^2/100$
$v = 22$ m/s

2001: A space odyssey



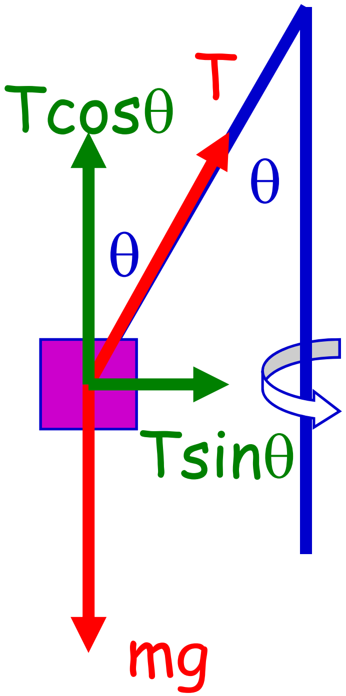


A space ship rotates with a linear velocity of 50 m/s. What should the distance from the central axis to the crew's cabin's be so that the crew feels like they are on earth? (the floor of the cabins is the inside of the outer edge of the spaceship)

The rotating spaceship has an acceleration directed towards the center of the ship: the 'lack' of forces acting on the crew pushes them against the ship.

$\Sigma F=ma$ $mg=mv^2/r$ so $r=v^2/9.8$ and thus $r=255$ m

Conical motion



What is the centripetal acceleration if the mass is 1 kg and $\theta=20^o$?

Vertical direction:

$\Sigma F=ma$

$T\cos\theta - mg=0$

So $T=mg/\cos\theta$

Horizontal direction:

$\Sigma F=ma_c$

$T\sin\theta=ma_c$

$mg\sin\theta/\cos\theta=mg\tan\theta=ma_c$

$a_c=g\tan\theta=9.8*0.36=3.6\ m/s^2$

A general strategy

- As usual, make a drawing of the problem, if not given.
- Draw all the forces that are acting on the object(s) under investigation.
- Decompose each of these into directions toward the center of the circular path and perpendicular to it.
- Realize that $\Sigma F_{\text{to center}} = m a_c = m v^2/r$

The gravitational force, revisited

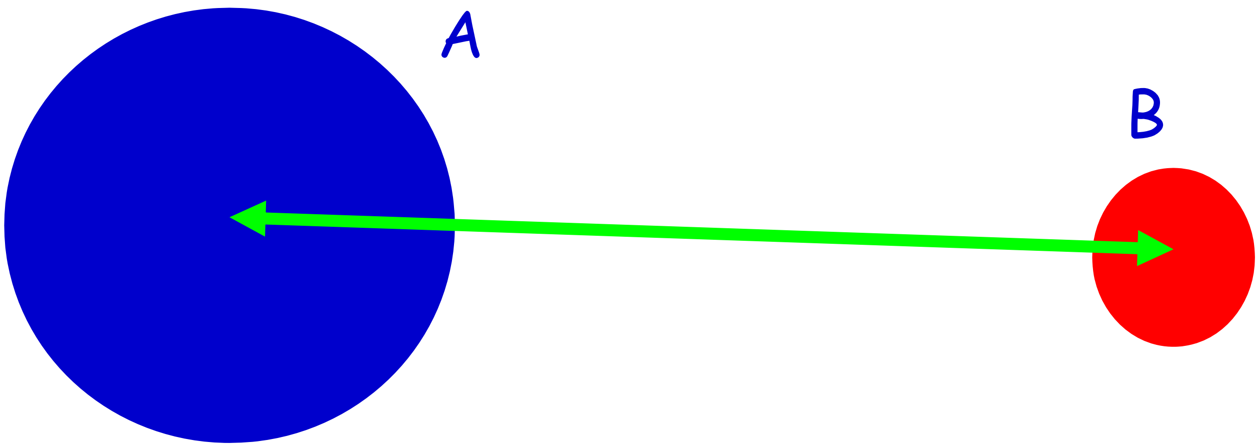
Newton:

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

$G=6.673\cdot10^{-11}$ N m^2/kg^2

The gravitational force works between every two massive particles in the universe, yet is the least well understood force known.

Gravitation between two objects



The gravitational force exerted by the spherical object A on B can be calculated by assuming that all of A's mass would be concentrated in its center and likewise for object B.

Conditions: B must be outside of A

A and B must be 'homogeneous'

Gravitational acceleration

$$F = G\frac{m_1 m_{EARTH}}{r^2}$$

F=mg

$g=Gm_{EARTH}/r^2$

On earth surface: $g=9.81$ m/s^2 $r=6366$ km

On top of mount Everest: $r=6366+8.850$ km $g=9.78$ m/s^2

Low-orbit satellite: $r=6366+1600$km $g=6.27$ m/s^2

Geo-stationary satellite: $r=6366+36000$ km

$g=0.22$ m/s^2

Losing weight easily?

What is your weight in a spaceship orbiting the earth?

Weightless!

Why?

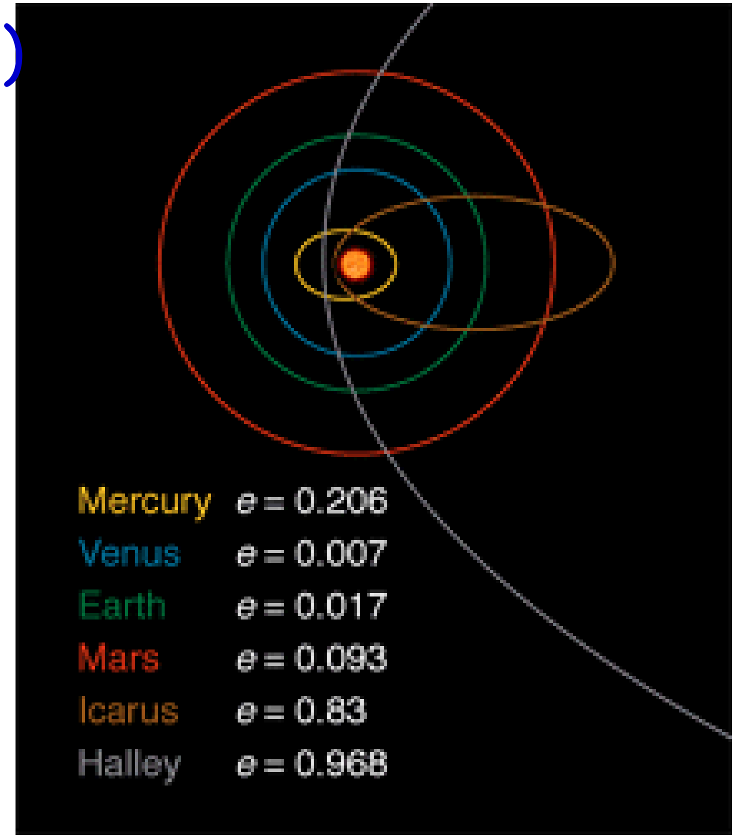
You and the spaceship 'fall' towards the earth by the same amount (same centripetal acceleration!) during every time interval.

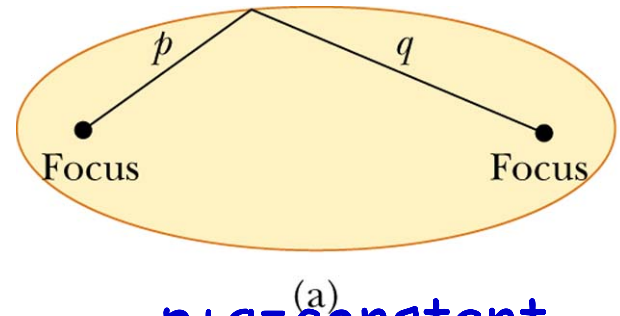
Kepler's laws

Johannes Kepler
(1571-1630)

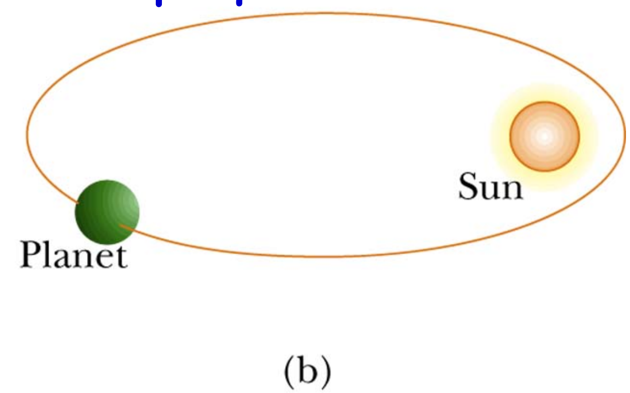
Kepler's First law

Ellipticity e(0-1)



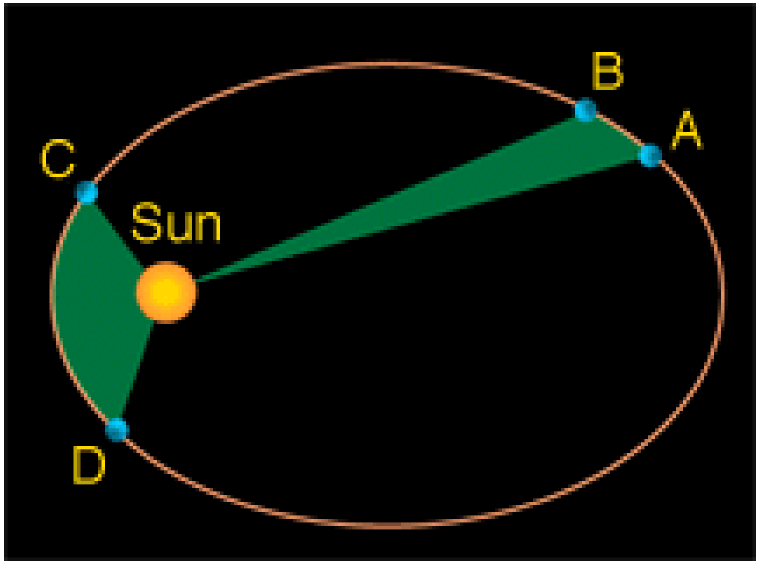


p+q=constant



An object A bound to another object B by a force that goes with $1/r^2$ moves in an elliptical orbit around B, with B being in one of the focus point of the ellipse; planets around the sun.

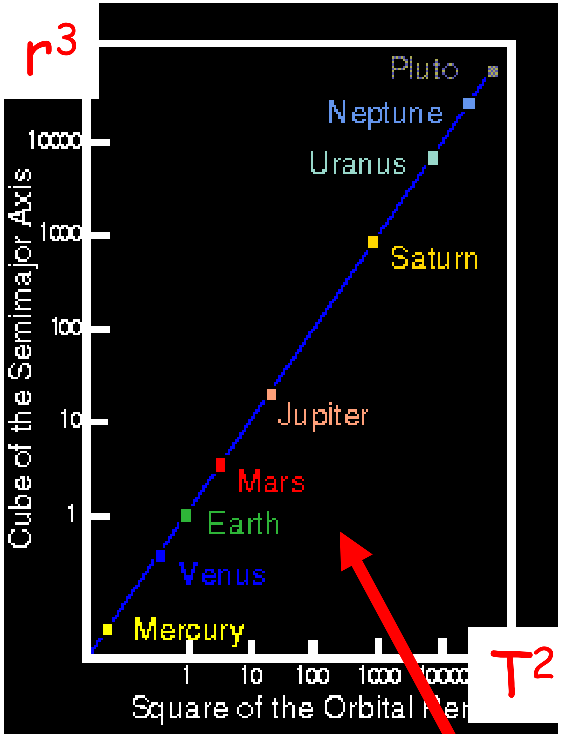
Kepler's second law



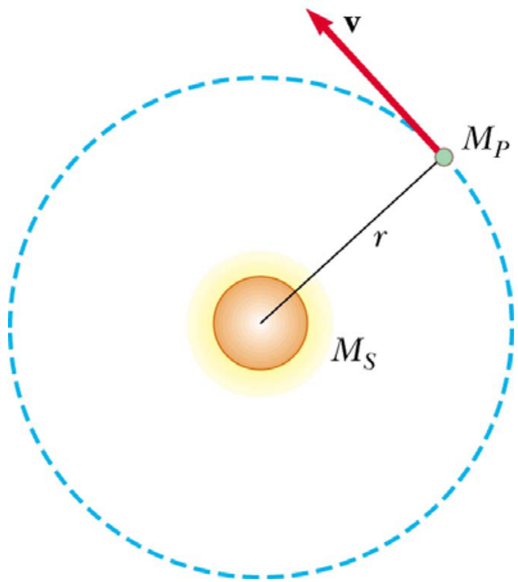
Area(D-C-SUN)=Area(B-A-SUN)

A line drawn from the sun to the elliptical orbit of a planet sweeps out equal areas in equal time intervals.

Kepler's third law



Consider a planet in circular motion around the sun:



$$G\frac{M_{sun}M_{planet}}{r^2} = \frac{M_{planet}v_{planet}^2}{r}$$

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T}$$

$$T^2 = \left(\frac{4\pi^2}{GM_{sun}}\right) r^3 = K_s r^3$$

$$K_s = 2.97\cdot 10^{-19}\ s^2/m^3$$

T: period-time it takes to make one revolution

$r^3 = T^2/K_s$ $\quad$ r^3=constant*T^2