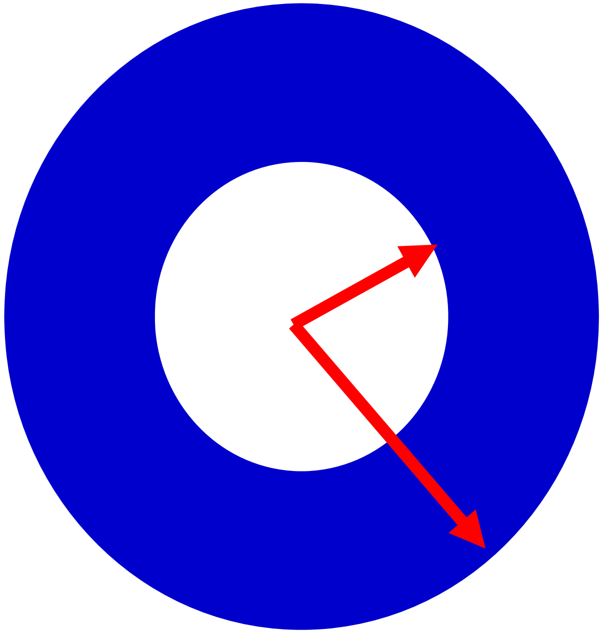
Thermal expansion: an example

In the early morning (T=30° F=272.4 K) a person is asked to measure the length of a football field with an aluminum measure and finds 109.600 m. Another person does the same in the afternoon (T=60° F=289.1 K) using the same ruler and finds 109.566 m. What is the coefficient of linear expansion of the ruler?

$\Delta L = \alpha L_o \Delta T$ so $\alpha = \Delta L/(L_0 \Delta T)$
$\Delta T = 16.7$ K $L_0 = 109.60$ $\Delta L = 109.600 - 109.566 = 0.034$
So: $\alpha = 24E-06$ /K



A heated ring

A metal ring is heated. What is true:

a) The inside and outside radii become larger
b) The inside radius becomes larger, the outside radius becomes smaller
c) The inside radius becomes smaller, the outside radius becomes larger
d) The inside and outside radii become smaller

Weight of 1 mol of atoms

$\boxed{\text{1 mol of atoms: } A \text{ gram (} A\text{: mass number)}}$

Example: 1 mol of Carbon = 12 g
1 mol of Zinc = 65.4 g

What about molecules?
H_2O 1 mol of water molecules:
2x 1 g (due to Hydrogen)
1x 16 g (due to Oxygen)
Total: 18 g

Example

A cube of silicon (molar mass 28.1 g) is 250 g.
A) How much silicon atoms are in the cube?
B) What would be the mass for the same number of gold atoms (molar mass 197 g)

A) Total number of mol: 250/28.1 = 8.90 mol
8.9 mol x 6.02×10^{23} particles = 5.4×10^{24} atoms

B) 8.90 mol x 197 g = 1.75×10^{3} g

Example

An ideal gas occupies a volume of 1.0 cm^3 at 20 ^0C at 1 atm.

A) How many molecules are in the volume?

B) If the pressure is reduced to 1.0×10^{-11} Pa, while the temperature drops to 0 ^0C, how many molecules remained in the volume?

A) $PV/T=nR$, so $n=PV/(TR)$ $R=8.31$ J/mol.K

$T=20\ ^0C=293$ K $P=1$ atm$=1.013\times10^5$ Pa $V=1.0\ cm^3=1\times10^{-6}\ m^3$

$n=4.2\times10^{-5}$ mol $n=4.2\times10^{-5}*N_A=2.5\times10^{19}$ molecules

B) $T=0\ ^0C=273$ K $P=1.0\times10^{-11}$ Pa $V=1\times10^{-6}\ m^3$

$n=4.4\times10^{-21}$ mol $n=2.6\times10^3$ particles (almost vacuum)

And another!

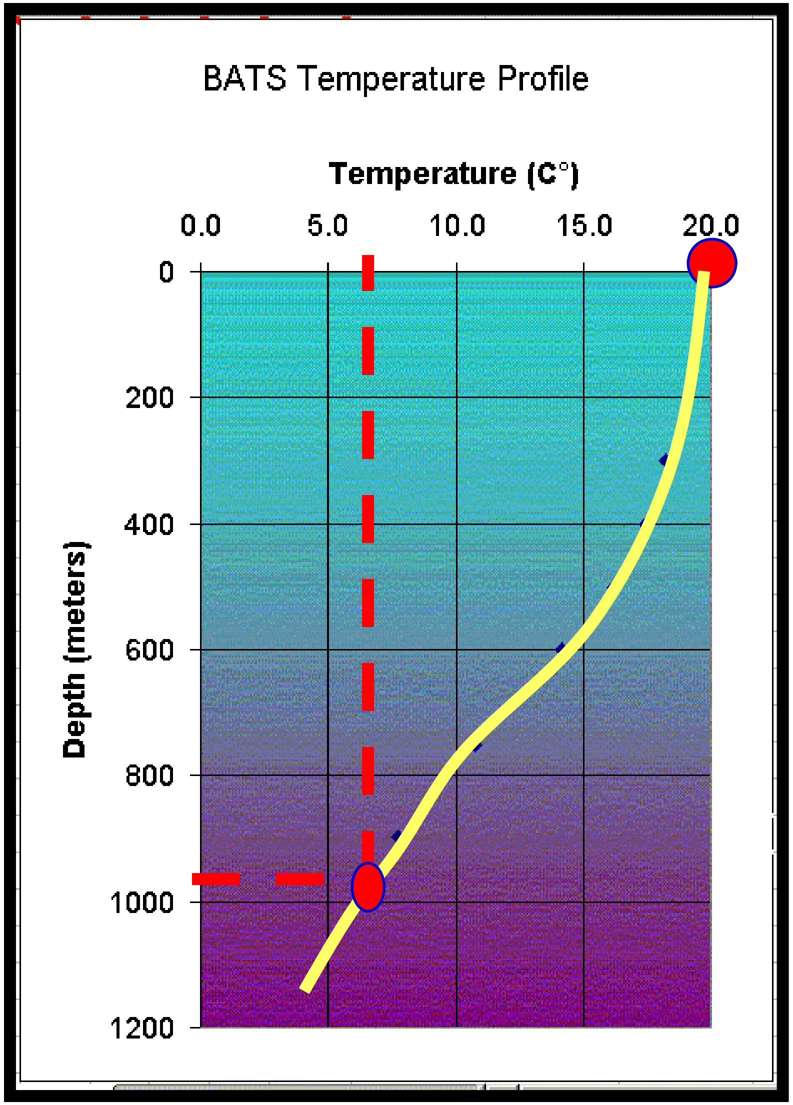
An air bubble has a volume of 1.50 cm^3 at 950 m depth (T=7 °C). What is its volume when it reaches the surface (ρ_{water}=1.0x10^3 kg/m^3)?

$P_{950m}=P_0+\rho_{water}gh$
$=1.013\times10^5+1.0\times10^3\times950\times9.81$
$=9.42\times10^6$ Pa

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

$$\frac{9.42\cdot10^6\times1.50\cdot10^{-6}}{280}=\frac{1.013\cdot10^5\times V_{surface}}{293}$$

$V_{surface}=1.46\times10^{-4}\ m^3=146\ cm^3$



Correlations

A volume with dimensions L x W x H is kept under pressure P at temperature T. A) If the temperature is raised by a factor of 2, and the height of the volume made 5 times smaller, by what factor does the pressure change?

Use the fact PV/T is constant if no gas is added/leaked

$P_1V_1/T_1 = P_2V_2/T_2$

$P_1V_1/T_1 = P_2(V_1/5)/(2T_1)$

$P_2=5*2*P_1=10P_1$

A factor of 10.

Diving Bell

A cylindrical diving bell (diameter 3 m and 4 m tall, with an open bottom is submerged to a depth of 220 m in the sea. The surface temperature is 25 ^0C and at 220 m, $T=5\ ^0C$. The density of sea water is 1025 kg/m^3. How high does the sea water rise in the bell when it is submerged?

Consider the air in the bell.

$P_{surf}=1.0\times10^5$ Pa $V_{surf}=\pi r^2h=28.3\ m^3$ $T_{surf}=25+273=298$ K

$P_{sub}=P_0+\rho_w g*depth=2.3\times10^6$ Pa $V_{sub}=?$ $T_{sub}=5+273=278$ K

Next, use PV/T=constant

$P_{surf}V_{surf}/T_{surf}=P_{sub}V_{sub}/T_{sub}$ plug in the numbers and find:

$V_{sub}=1.15\ m^3$ (amount of volume taken by the air left)

$V_{taken\ by\ water}=28.3-1.15=27.15\ m^3=\pi r^2h$

$h=27.15/\pi r^2=3.8$ m rise of water level in bell.

example

What is the rms speed of air at 1 atm and room temperature? Assume it consist of molecular nitrogen only (N_2)?

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_bT}{m}} = \sqrt{\frac{3RT}{M}}$$

R=8.31 J/mol K T=293 K M=2*14x10^{-3} kg/mol

v_{rms}=511 m/s !!!!!

And another...

What is the total kinetic energy of the air molecules in the lecture room (assume only molecular nitrogen is present N_2)?

1) find the total number of molecules in the room

$PV/T = Nk_b$ $\quad P=1.015\times10^5$ Pa $\quad V=10*4*25=1000\ m^3$

$k_b=1.38\times10^{-23}$ J/K $\quad T=293$ K

$N=2.5\times10^{28}$ molecules (4.2×10^4 mol)

2) $E_{kin}=(3/2)Nk_BT=1.5\times10^8$ J

(same as driving a 1000 kg car at 547.7 m/s)