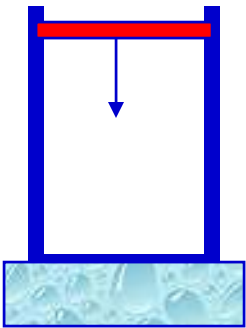


In which case is the work done on the gas largest?

The area under the curves in cases B and C is largest (i.e. the absolute amount of work is largest). In case C, the volume becomes larger and the pressure lower (the piston is moved up) so work is done **by** the gas (work done on the gas is negative). In case B the work done **on** the gas is positive, and thus largest.

# First law: examples 1) isobaric process

A gas in a cylinder is kept at  $1.0 \times 10^5 \text{ Pa}$ . The cylinder is brought in contact with a cold reservoir and 500 J of heat is extracted. Meanwhile the piston has sunk and the volume changed by  $100 \text{ cm}^3$ . What is the change in internal energy?



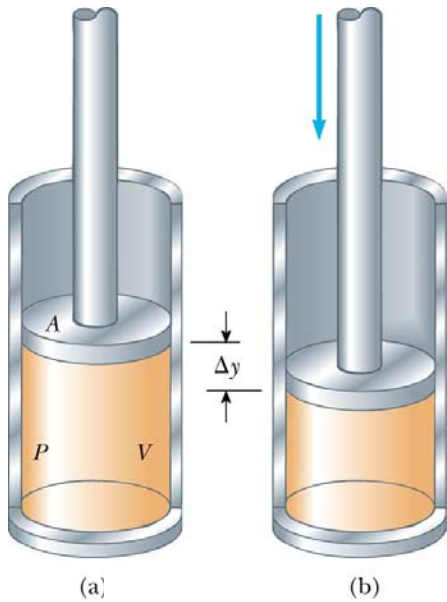
$$Q = -500 \text{ J}$$

$$W = -P\Delta V = -1.0 \times 10^5 \times -100 \times 10^{-6} = 10 \text{ J}$$

$$\Delta U = Q + W = -500 + 10 = -490 \text{ J}$$

In an isobaric process both  $Q$  and  $W$  are non-zero.

## First Law: examples: 2) Adiabatic process



A piston is pushed down rapidly. Because the transfer of heat through the walls takes a long time, no heat can escape.

During the motion of the piston, the temperature has risen  $100\text{ }^{\circ}\text{C}$ . If the container has 10 moles of an ideal gas, how much work has been done during the compression?

**Adiabatic: No heat transfer,  $Q=0$**

$$\Delta U = Q + W = W$$

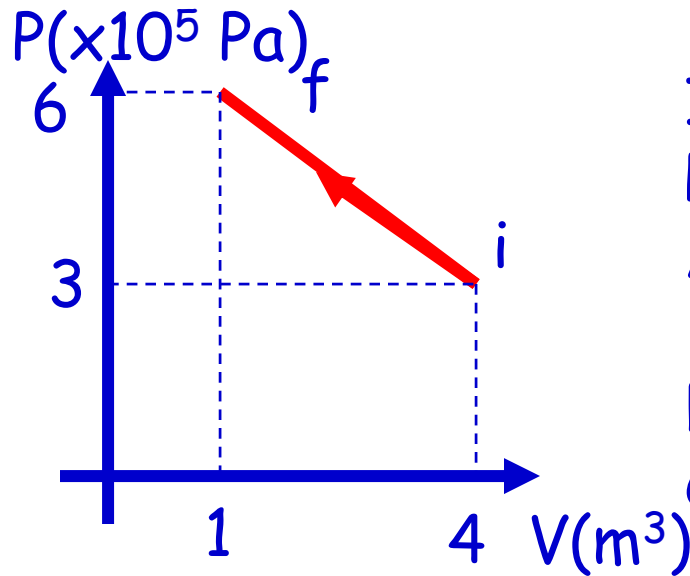
$$\Delta U = \left(\frac{3}{2}\right)nR\Delta T = \left(\frac{3}{2}\right) \times 10 \times 8.31 \times 100 = 12465\text{ J}$$

(ideal gas: internal energy is kinetic energy  $U = \left(\frac{3}{2}\right)nRT$ )

12465 J of work has been done on the gas.

Why can we not use  $W = -P\Delta V$ ???

# First Law: examples 3) general case



In ideal gas is compressed (see P-V diagram).

A) What is the change in internal energy

b) What is the work done on the gas?

C) How much heat has been transferred to the gas?

A)  $U = (3/2)nRT$  and  $PV = nRT$  so,  $U = (3/2)PV$  &  $\Delta U = (3/2)\Delta(PV)$

$$\Delta U = 3/2(P_f V_f - P_i V_i) = 3/2[(6E+05) \times 1 - (3E+05) \times 4] = -9E+5 \text{ J}$$

B) Work: area under the P-V graph:  $(9+4.5) \times 10^5 = 13.5 \times 10^5$   
(positive since work is done on the gas)

C)  $\Delta U = Q + W$  so  $Q = \Delta U - W = (-9E+5) - (13.5E+5) = -22.5E+5 \text{ J}$   
Heat has been extracted from the gas.

# Cyclic Process, step by step 1

Process A-B.

Negative work is done on the gas:  
(the gas is doing positive work).

$W = -\text{Area under P-V diagram}$



$$\begin{aligned} &= -[(50-10) \cdot 10^{-3}] \cdot [(1.0-0.0) \cdot 10^5] \\ &\quad - \frac{1}{2} [(50-10) \cdot 10^{-3}] \cdot [(5.0-1.0) \cdot 10^5] = \\ &= 4000 + 8000 \end{aligned}$$

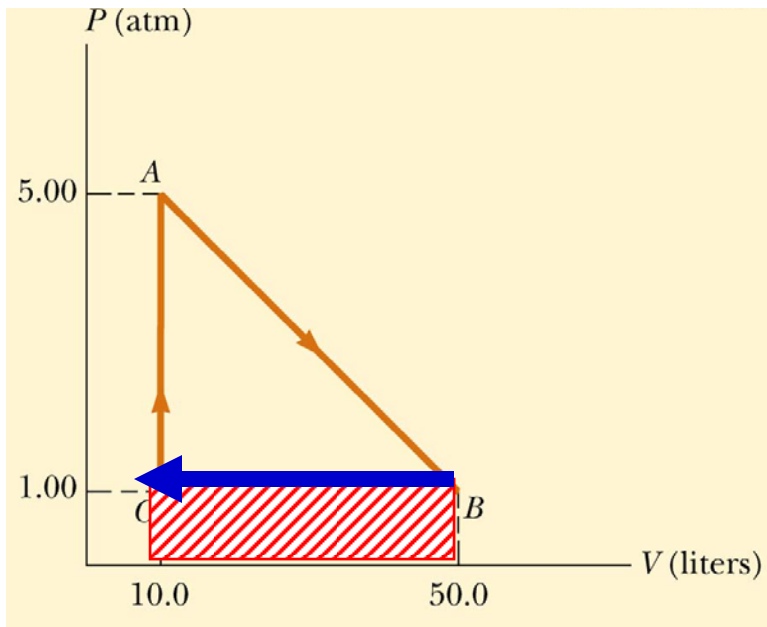
$$W = -12000 \text{ J}$$

$$\begin{aligned} \Delta U &= \frac{3}{2} n R \Delta T = \frac{3}{2} (P_B V_B - P_A V_A) = \\ &= 1.5 \cdot [(1 \text{E}+5)(50 \text{E}-03) - (5 \text{E}+5)(10 \text{E}-03)] = 0 \end{aligned}$$

The internal energy has not changed

$\Delta U = Q + W$  so  $Q = \Delta U - W = 12000 \text{ J}$ : Heat that was added to the system was used to do the work!

## Cyclic process, step by step 2



Process B-C

$W = \text{Area under P-V diagram}$



$$= [(10-50) \times 10^{-3} \times (1.0-0.0) \times 10^5] =$$

$$W = 4000 \text{ J}$$

Work was done on the gas

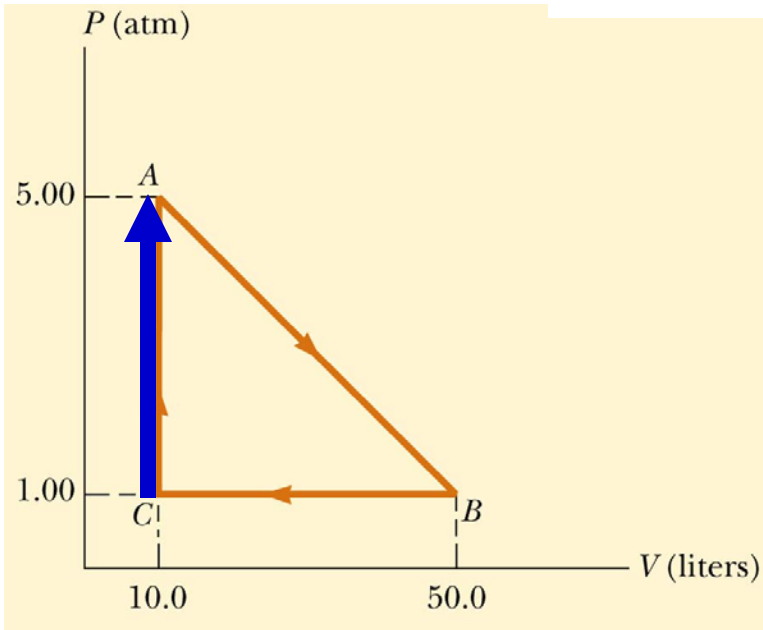
$$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}(P_c V_c - P_b V_b) =$$
$$= 1.5[(1 \times 10^5)(10 \times 10^{-3}) - (1 \times 10^5)(50 \times 10^{-3})] = -6000 \text{ J}$$

The internal energy has decreased by 6000 J

$$\Delta U = Q + W \text{ so } Q = \Delta U - W = -6000 - 4000 \text{ J} = -10000 \text{ J}$$

10000 J of energy has been transferred out of the system.

## Cyclic process, step by step 3



Process C-A

$W = -\text{Area under P-V diagram}$

$$W = 0 \text{ J}$$

No work was done on/by the gas.

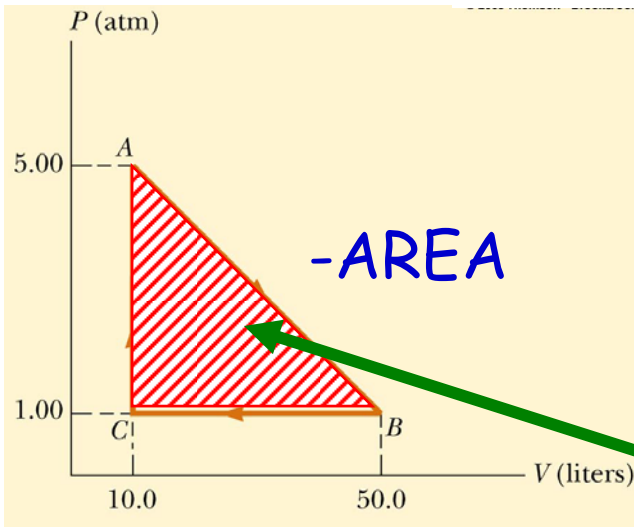
$$\begin{aligned}\Delta U &= \frac{3}{2}nR\Delta T = \frac{3}{2}(P_c V_c - P_b V_b) = \\ &= 1.5[(5 \times 10) - (1 \times 50)] = +6000 \text{ J}\end{aligned}$$

The internal energy has increased by 6000 J

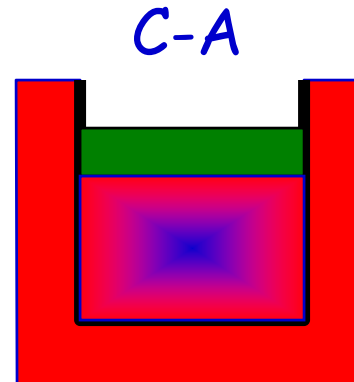
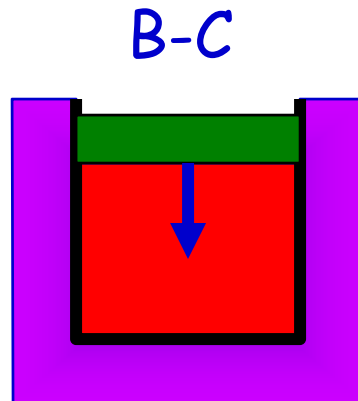
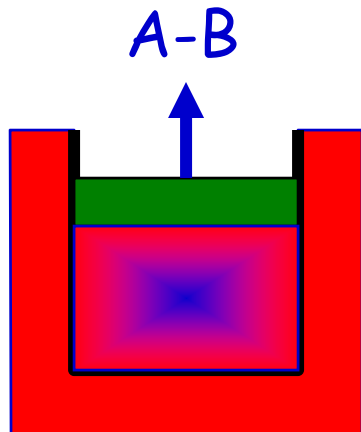
$$\Delta U = Q + W \text{ so } Q = \Delta U - W = 6000 - 0 \text{ J} = 6000 \text{ J}$$

6000 J of energy has been transferred into the system.

# Summary of the process

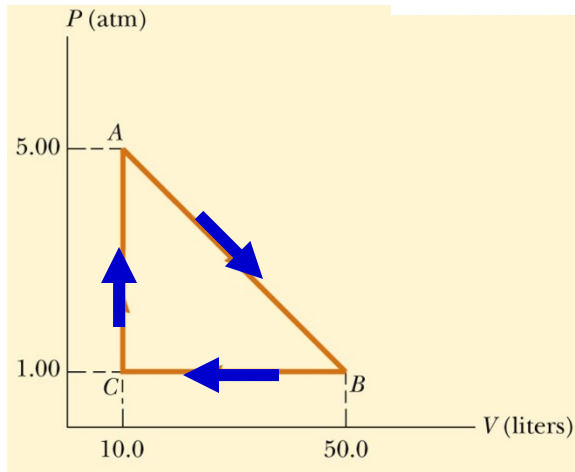


Quantity Process	Work(W)	Heat(Q)	$\Delta U$
A-B	-12000 J	12000 J	0
B-C	4000 J	-10000 J	-6000
C-A	0 J	6000 J	6000
SUM	-8000 J	8000 J	0



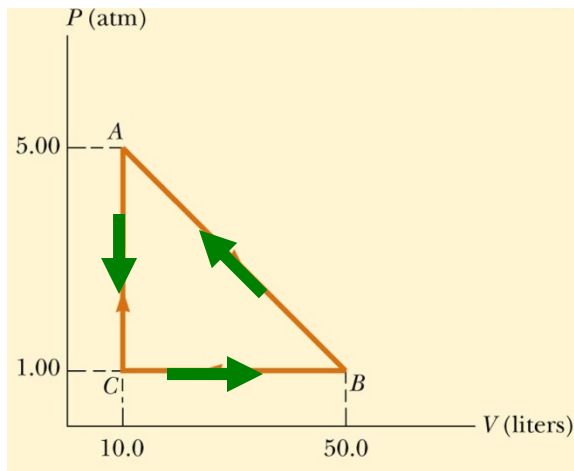


# What did we do?



The gas performed net work (8000 J) while heat was supplied (8000 J):

**We have built an engine!**



What if the process was done in the reverse way?

Net work was performed on the gas and heat extracted from the gas.

**We have built a heat pump!  
(A fridge)**

# Examples

One mole of an ideal gas initially at  $0\text{ }^{\circ}\text{C}$  undergoes an expansion at constant pressure of one atmosphere to four times its original volume.

- What is the new temperature?
- What is the work done on the gas?

a)  $PV/T = \text{constant}$  so if  $V \times 4$  then  $T \times 4$   $273\text{K} \times 4 = 1092\text{ K}$

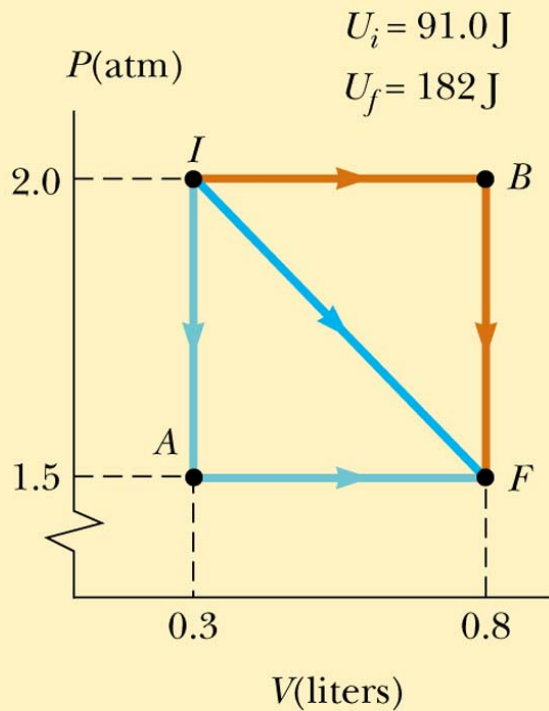
b)  $W = -P\Delta V$

use  $PV = nRT$

before expansion:  $PV = 1 \times 8.31 \times 273 = 2269\text{ J}$

after expansion:  $PV = 1 \times 8.31 \times 1092 = 9075\text{ J}$

$W = -P\Delta V = -\Delta(PV) = -[(PV)_f - (PV)_i] = -(9075 - 2269) = -6806\text{ J}$   
-6805 J of work is done on the gas.



## Example

A gas goes from initial state I to final state F, given the parameters in the figure. What is the work done on the gas and the net energy transfer by heat to the gas for:

a) path IBF   b) path IF   c) path IAF  
 $(U_i = 91 \text{ J} \quad U_f = 182 \text{ J})$

a) work done: area under graph:

$$W = -(0.8 - 0.3) \cdot 10^{-3} \cdot 2.0 \cdot 10^5 = -100 \text{ J}$$

$$\Delta U = W + Q \quad 91 = -100 + Q \quad \text{so } Q = 191 \text{ J}$$

b)  $W = -[(0.8 - 0.3) \cdot 10^{-3} \cdot 1.5 \cdot 10^5 + \frac{1}{2}(0.8 - 0.3) \cdot 10^{-3} \cdot 0.5 \cdot 10^5] = -87.5 \text{ J}$

$$\Delta U = W + Q \quad 91 = -87.5 + Q \quad \text{so } Q = 178.5 \text{ J}$$

c)  $W = -[(0.8 - 0.3) \cdot 10^{-3} \cdot 1.5 \cdot 10^5] = -75 \text{ J}$

$$\Delta U = W + Q \quad 91 = -75 + Q \quad \text{so } Q = 166 \text{ J}$$

## Example

The efficiency of a Carnot engine is 30%. The engine absorbs 800 J of energy per cycle by heat from a hot reservoir at 500 K. Determine a) the energy expelled per cycle and b) the temperature of the cold reservoir. c) How much work does the engine do per cycle?

a) Generally for an engine: efficiency:  $1 - |Q_{\text{cold}}| / |Q_{\text{hot}}|$   
 $0.3 = 1 - |Q_{\text{cold}}| / 800$ , so  $|Q_{\text{cold}}| = -(0.3 - 1) * 800 = 560 \text{ J}$

b) for a Carnot engine: efficiency:  $1 - T_{\text{cold}} / T_{\text{hot}}$   
 $0.3 = 1 - T_{\text{cold}} / 500$ , so  $T_{\text{cold}} = -(0.3 - 1) * 500 = 350 \text{ K}$

c)  $W = |Q_{\text{hot}}| - |Q_{\text{cold}}| = 800 - 560 = 240 \text{ J}$

## A new powerplant

A new powerplant is designed that makes use of the temperature difference between sea water at 0 m ( $20^{\circ}$ ) and at 1 km depth ( $5^{\circ}$ ). A) what would be the maximum efficiency of such a plant? B) If the powerplant produces 75 MW, how much energy is absorbed per hour? C) Is this a good idea?

a) maximum efficiency=Carnot efficiency= $1-T_{\text{cold}}/T_{\text{hot}}=1-278/293=0.051$  efficiency=5.1%

b)  $P=75 \times 10^6 \text{ J/s}$   $W=P \cdot t=75 \times 10^6 \cdot 3600=2.7 \times 10^{11} \text{ J}$   
efficiency= $1-|Q_{\text{cold}}|/|Q_{\text{hot}}|=(|Q_{\text{hot}}|-|Q_{\text{cold}}|)/|Q_{\text{hot}}|=W/|Q_{\text{hot}}|$  so  $|Q_{\text{hot}}|=W/\text{efficiency}=5.3 \times 10^{12} \text{ J}$

c) Yes! Very Cheap!! but...  $|Q_{\text{cold}}|=|Q_{\text{hot}}|-W=5.0 \times 10^{12} \text{ J}$   
every hour  $5 \times 10^{12} \text{ J}$  of waste heat is produced:  
 $Q=cm\Delta T$   $5 \times 10^{12}=4186 \cdot m \cdot 1$   $m=1 \times 10^9 \text{ kg}$  of water is heated by  $1^{\circ}\text{C}$ .

## Example

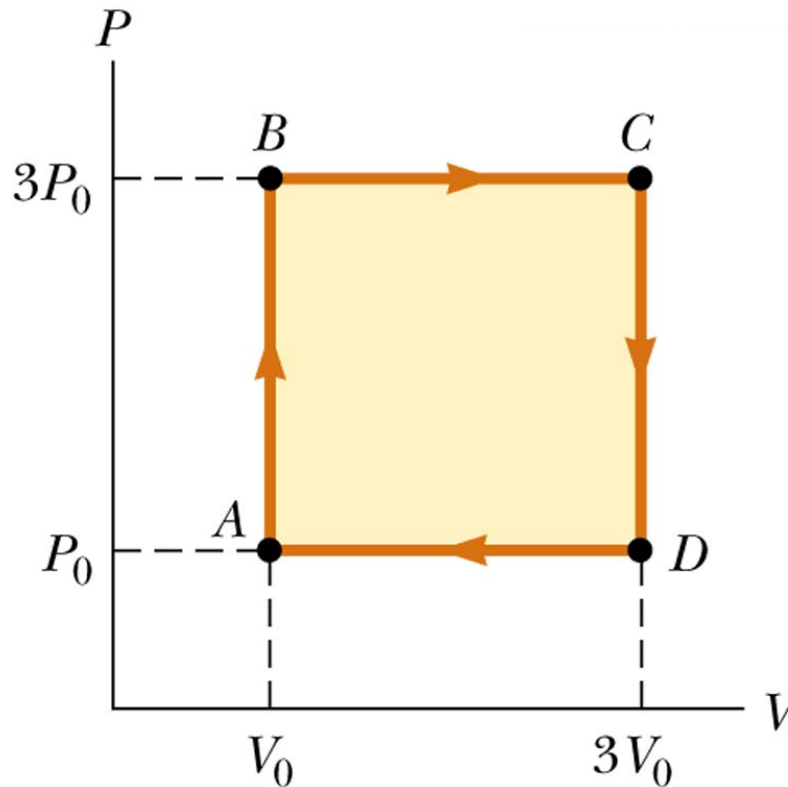
What is the change in entropy of 1.00 kg of liquid water at 100 °C as it changes to steam at 100 °C?

$$L_{\text{vaporization}} = 2.26\text{E}+6 \text{ J/kg}$$

$$Q = L_{\text{vaporization}} m = 2.26\text{E}+6 \text{ J/kg} * 1 \text{ kg} = 2.26\text{E}+6 \text{ J}$$

$$\Delta S = Q/T = 2.26\text{E}+6 / (373) = 6059 \text{ J/K}$$

## A cycle



- Consider the cycle in the figure.
- A) what is the net work done in one cycle?
- B) What is the net energy added to the system per cycle?

A) Work: area enclosed in the cycle:

$W = -(2V_0 \cdot 2P_0) + (2V_0 P_0) = -2V_0 P_0$  (Negative work is done on the gas, positive work is done by the gas)

b) Cycle:  $\Delta U = 0$  so  $Q = -W$   $Q = 2V_0 P_0$  of heat is added to the system.

## adiabatic process

For an adiabatic process, which of the following is true?

- A)  $\Delta S < 0$
- B)  $\Delta S = 0$
- C)  $\Delta S > 0$
- D) none of the above

Adiabatic:  $Q=0$  so  $\Delta S=Q/T=0$