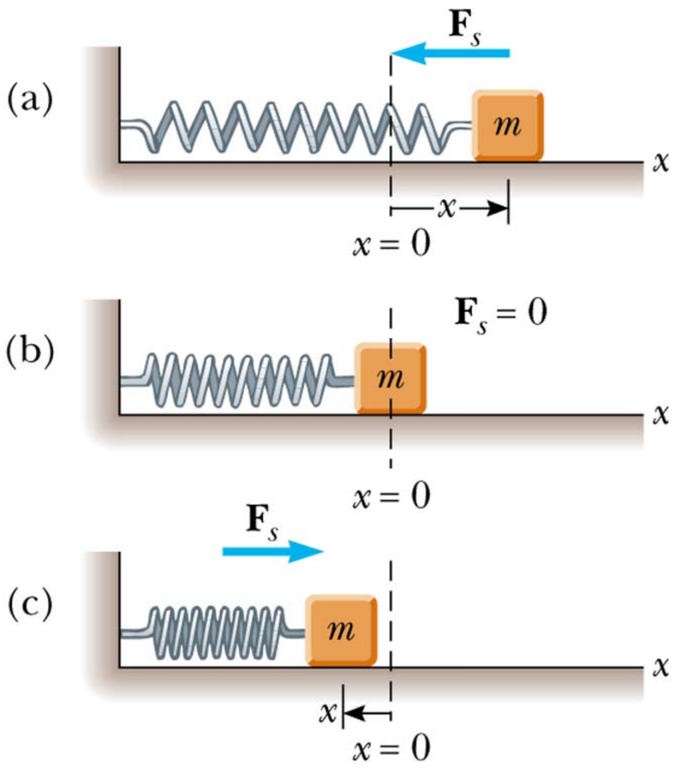
Hooke's law

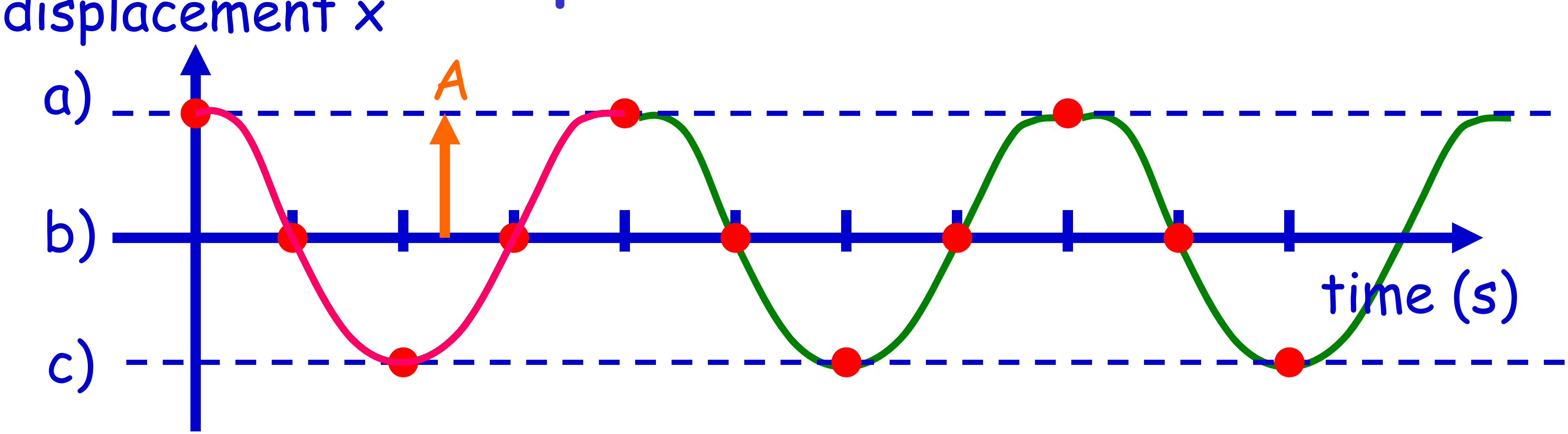


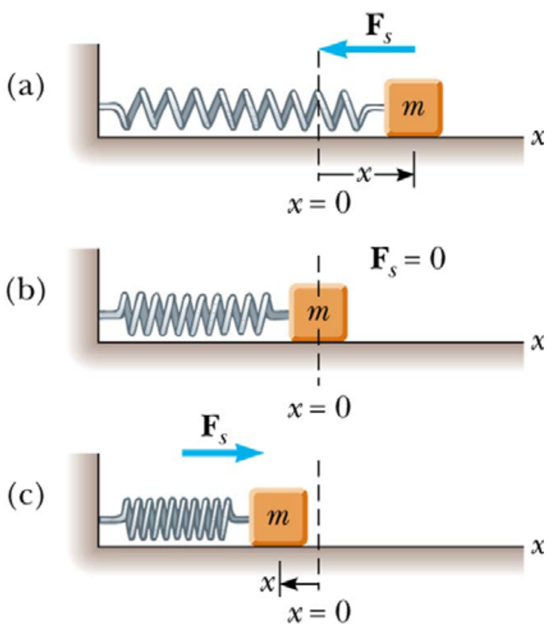
$F_s = -kx$ Hooke's law

If there is no friction, the mass continues to oscillate back and forth.

If a force is proportional to the displacement x, but opposite in direction, the resulting motion of the object is called: simple harmonic oscillation

Simple harmonic motion





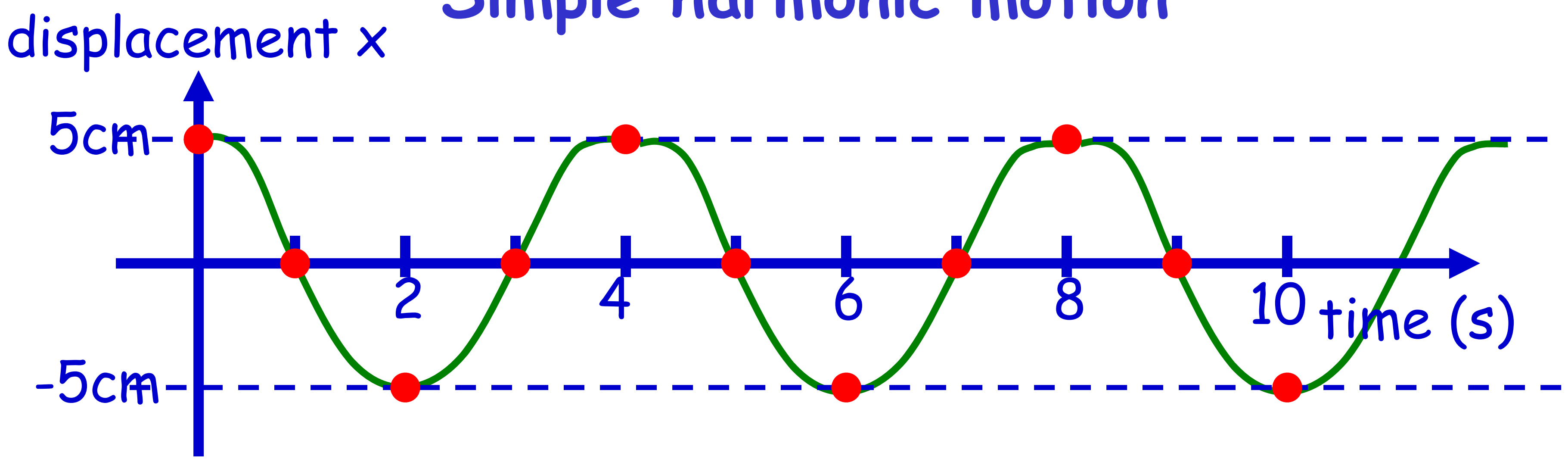
Amplitude (A): maximum distance from equilibrium (unit: m)

Period (T): Time to complete one full oscillation (unit: s)

Frequency (f): Number of completed oscillations per second (unit: 1/s = 1 Herz [Hz])

$$f=1/T$$

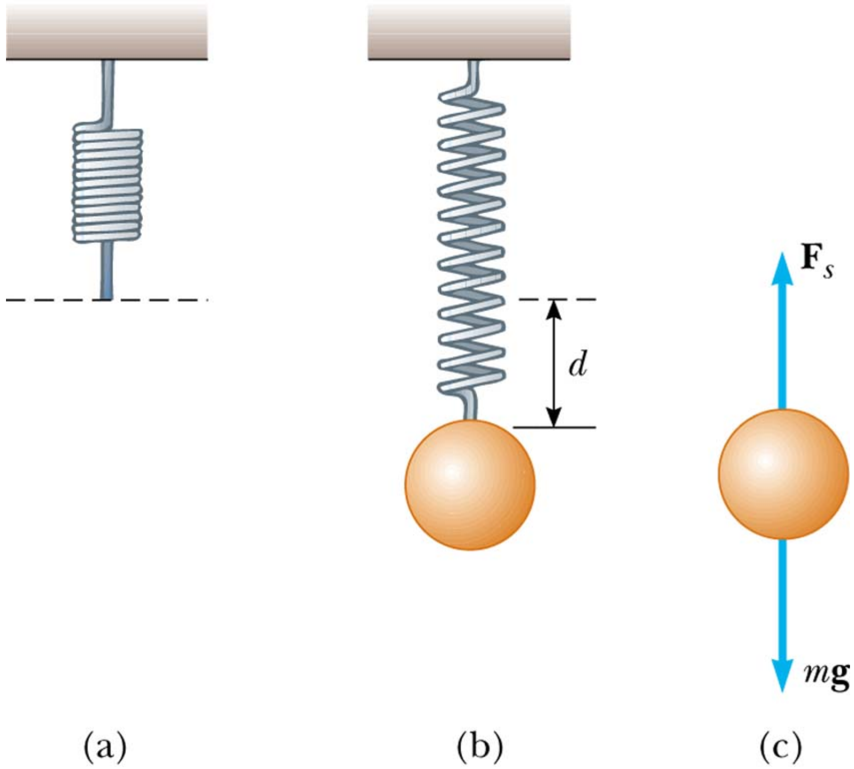
Simple harmonic motion



a) what is the amplitude of the harmonic oscillation?
b) what is the period of the harmonic oscillation?
c) what is the frequency of the harmonic oscillation?

a) Amplitude: 5 cm (0.05 m)
b) period: time to complete one full oscillation: 4s
c) frequency: number of oscillations per second=1/T=0.25 s

The spring constant k



When the object hanging from the spring is not moving:

$F_{spring} = -F_{gravity}$

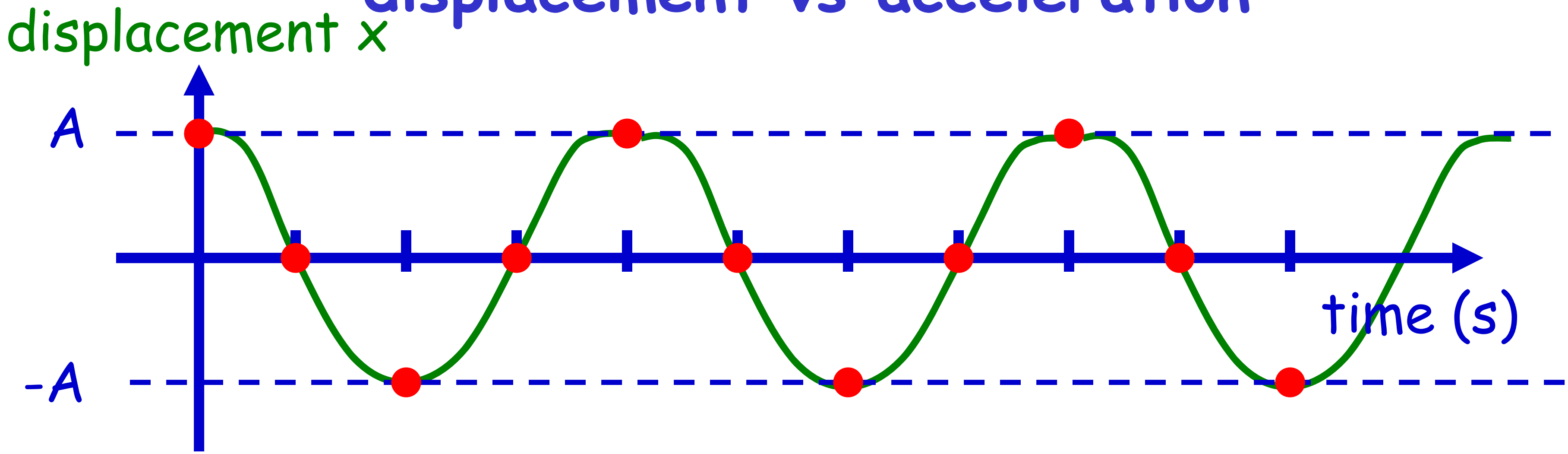
$-kd = -mg$

$k = mg/d$

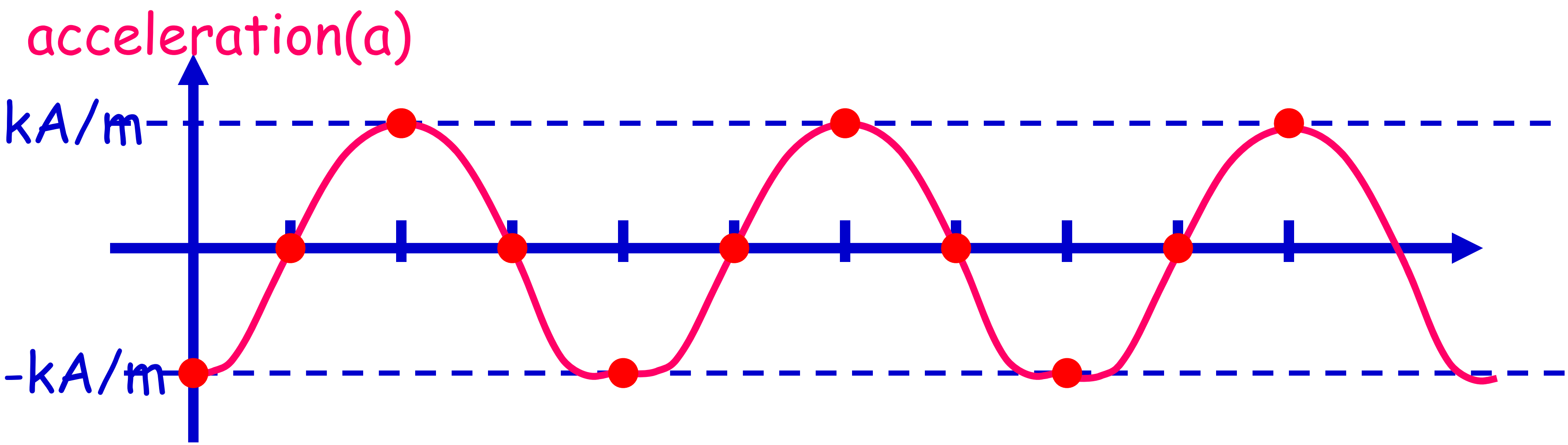
k is a constant, so if we hang twice the amount of mass from the spring, d becomes twice larger:

$k=(2m)g/(2d)=mg/d$

displacement vs acceleration



Newton's second law: $F=ma \Rightarrow -kx=ma \Rightarrow a=-kx/m$



example

A mass of 1 kg is hung from a spring. The spring stretches by 0.5 m. Next, the spring is placed horizontally and fixed on one side to the wall. The same mass is attached and the spring stretched by 0.2 m and then released. What is the acceleration upon release?

1st step: find the spring constant k

F_{spring} $=-F_{gravity}$ or $-kd = -mg$

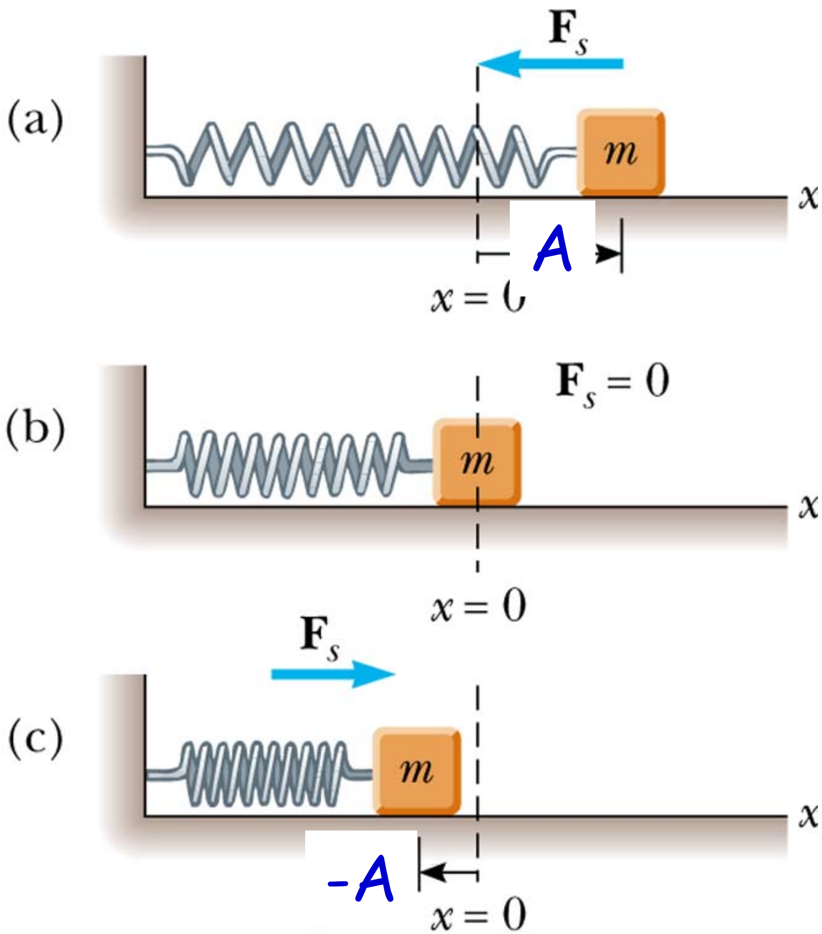
$k = mg/d = 1*9.8/0.5 = 19.6$ N/m

2nd step: find the acceleration upon release

Newton's second law: $F=ma \Rightarrow -kx=ma \Rightarrow a=-kx/m$

$a=-19.6*0.2/1=-3.92$ m/s^2

energy and velocity



	$E_{kin}(\frac{1}{2}mv^2)$	$E_{pot,spring}(\frac{1}{2}kx^2)$	Sum
(a)	0	$\frac{1}{2}kA^2$	$\frac{1}{2}kA^2$
(b)	$\frac{1}{2}mv^2$	0	$\frac{1}{2}mv^2$
(c)	0	$\frac{1}{2}k(-A)^2$	$\frac{1}{2}kA^2$

conservation of ME: $\frac{1}{2}m[v(x=0)]^2=\frac{1}{2}kA^2$ so $v(x=0)=\pm A\sqrt{(k/m)}$

velocity more general

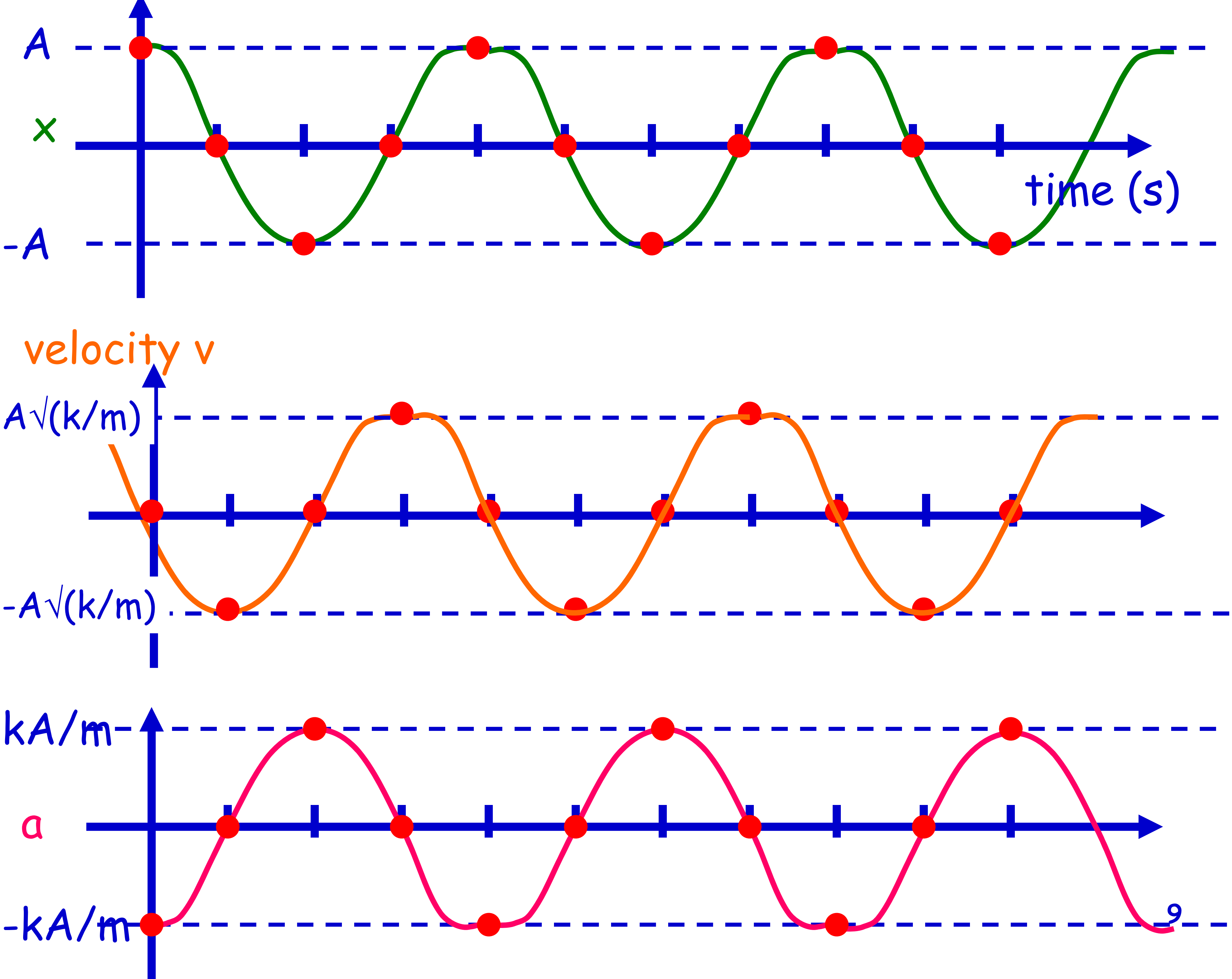
Total ME at any displacement x: $\frac{1}{2}mv^2+\frac{1}{2}kx^2$

Total ME at max. displacement A: $\frac{1}{2}kA^2$

Conservation of ME: $\frac{1}{2}kA^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2$

So: $v=\pm\sqrt{[(A^2-x^2)k/m]}$

position X	velocity V	acceleration a
$+A$	0	$-kA/m$
0	$\pm A\sqrt{(k/m)}$	0
$-A$	0	kA/m



A
x
-A
time (s)
velocity v
A√(k/m)
-A√(k/m)
kA/m
a
-kA/m

Generally: also add gravitational PE

$$ME = KE + PE_{spring} + PE_{gravity}$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2 + mgh$$

An example

A 0.4 kg object, connected to a light spring with a spring constant of 19.6 N/m oscillates on a frictionless horizontal surface. If the spring is compressed by 0.04 and then released determine: a) the maximum speed of the object b) the speed of the object when the spring is compressed by 0.015 m c) when it is stretched by 0.015 m d) for what value of x does the speed equal one half of the maximum speed?

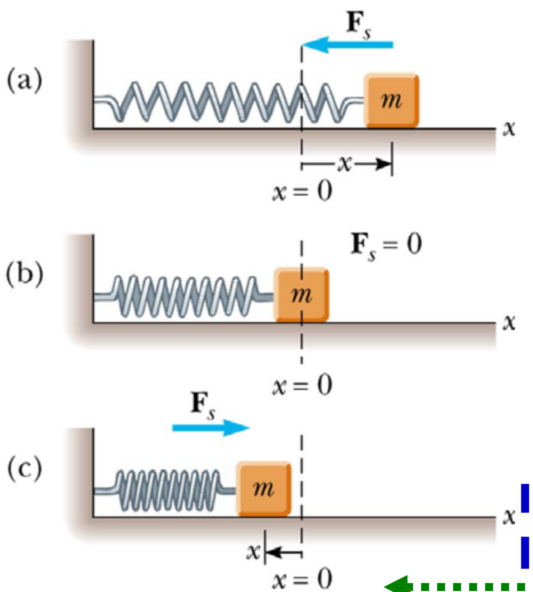
a) $v=\sqrt{[(A^2-x^2)k/m]}$ (speed is always positive!)
maximum if x=0: $\sqrt{[A^2k/m]}=0.04\sqrt{(19.6/0.4)}=0.28$ m/s

b) $v=\sqrt{[(A^2-x^2)k/m]}$ at $x=-0.015$
$v=\sqrt{[((0.04)^2-(-0.015)^2)19.6/0.4]}=0.26$ m/s

c) same as b)

d) $\sqrt{[(A^2-x^2)k/m]}=0.28/2=0.14$ $x=\sqrt{(A^2-0.14^2m/k)}=0.035$m

circular motion & simple harmonic motion

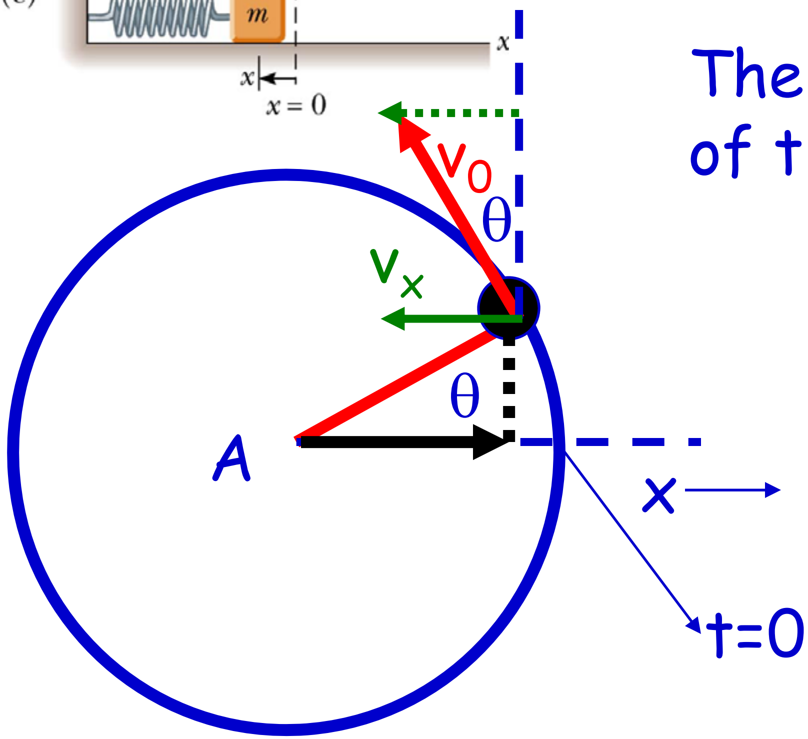


A particle moves in a circular orbit with angular velocity ω, corresponding to a linear velocity $v_0=\omega r=\omega A$

The horizontal position as a function of time: $x(t)=A\cos\theta=A\cos(\omega t)$ $(\theta=\omega t)$

The horizontal velocity as a function of time: $\sin\theta=-v_x/v_0$

$$v_x(t)=-v_0\sin\theta=-\omega A\sin(\omega t)$$



Time to complete one circle (I.e. one period T):

$T=2\pi A/v_0=2\pi A/\omega A=2\pi/\omega$

$\omega=2\pi/T=2\pi f$ (f: frequency)

ω: angular frequency

Circular motion and simple harmonic motion

The simple harmonic motion can be described by the projection of circular motion on the horizontal axis.

$x_{harmonic}(t)=A\cos(\omega t)$
$v_{harmonic}(t)=-\omega A\sin(\omega t)$

where A is the amplitude of the oscillation, and $\omega=2\pi/T=2\pi f$, where T is the period of the harmonic motion and $f=1/T$ the frequency.

For the case of a spring

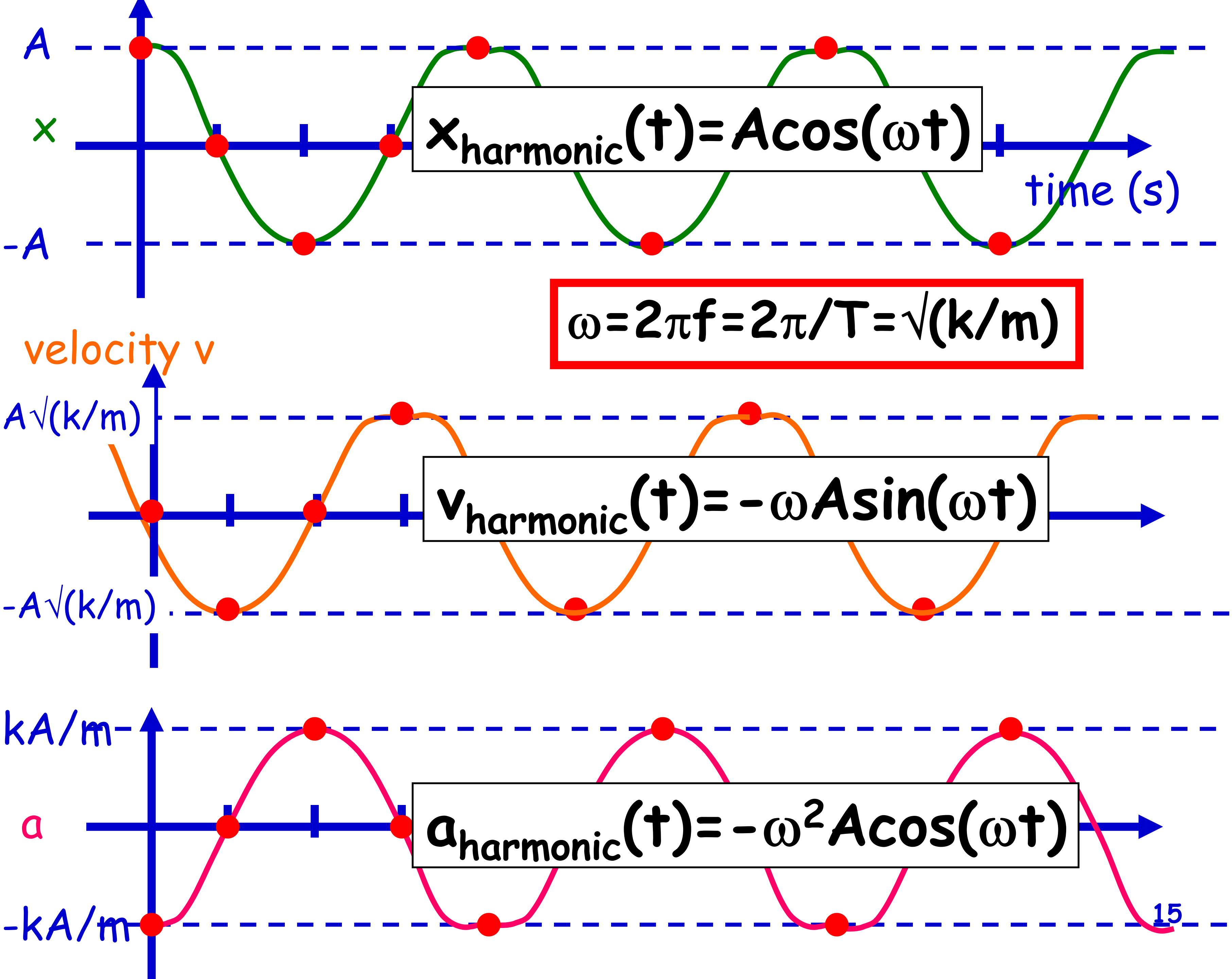
position X	velocity V	acceleration a
$+A$	0	$-kA/m$
0	$\pm A\sqrt{(k/m)}$	0
$-A$	0	kA/m

1) velocity is maximum if $v=\pm A\sqrt{(k/m)}$

2) circular motion: $v_{spring}(t)=-\omega A\sin\omega t$ maximal if $v_{spring}=\pm\omega A$

combine 1) & 2) $\boldsymbol{\omega=\sqrt{(k/m)}}$

Acceleration: $a(t)=-(kA/m)\cos(\omega t)=-\omega^2 A\cos(\omega t)$



$$x_{harmonic}(t) = A\cos(\omega t)$$

$$\omega = 2\pi f = 2\pi / T = \sqrt{(k/m)}$$

$$v_{harmonic}(t) = -\omega A \sin(\omega t)$$

$$a_{harmonic}(t) = -\omega^2 A\cos(\omega t)$$

Example

A mass of 0.2 kg is attached to a spring with k=100 N/m. The spring is stretched over 0.1 m and released.

a) What is the angular frequency (ω) of the corresponding circular motion?
b) What is the period (T) of the harmonic motion?
c) What is the frequency (f)?
d) What are the functions for x,v and t of the mass as a function of time? Make a sketch of these.

a) $\omega=\sqrt{(k/m)}=\ \omega=\sqrt{(100/0.2)}=22.4$ **rad/s**

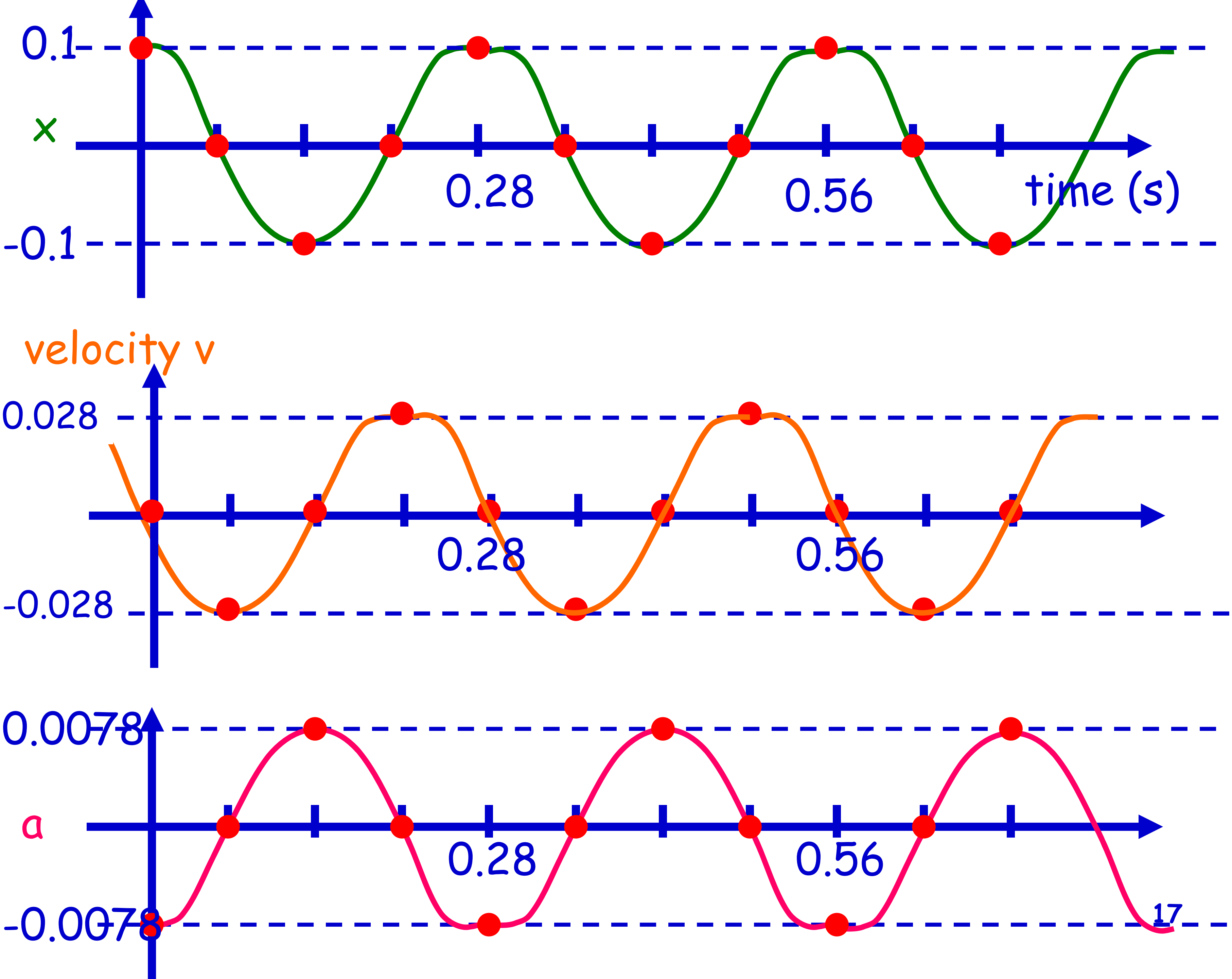
b) $\omega=2\pi/T \quad T=\ 2\pi/\omega=0.28$ **s**

c) $\omega=2\pi f \quad f=\omega/2\pi=3.55$ **Hz** $(=1/T)$

d) $x_{harmonic}(t)=A\cos(\omega t)=0.1\cos(0.28t)$

$v_{harmonic}(t)=-\omega A\sin(\omega t)=-0.028\sin(0.28t)$

$a_{harmonic}(t)=-\omega^2 A\cos(\omega t)=-0.0078\cos(0.28t)$



0.1
x
-0.1
0.28
0.56
time (s)
velocity v
0.028
-0.028
0.28
0.56
0.0078
a
-0.0078
0.28
0.56