GENERALIZING AREA DIFFERENCE BETWEEN IMAGES OF DOMAINS UNDER COMPLEX ANALYTIC MAPPINGS

Fock space, $\mathcal{F}^2(\mathbb{C}^n)$, consists of complex analytic functions on $\mathbb{C}^n$ such that their weighted square integral with respect to the weight $e^{-|z|^2}$ is finite. In $\mathbb{C}^n$, $D_j = \frac{\partial}{\partial z_j}$ is an unbounded linear operator on $\mathcal{F}^2(\mathbb{C}^n)$, known as *annihilator* operator, and M_k is the multiplication operator by z_k, known as *creation* operator. In fact, $D_j^* = M_j$ and $[D_j, M_k] = D_j M_k - M_k D_j = \delta_{jk}$. In quantum mechanics, **position** is defined by $A_j = \frac{D_j + M_j}{2}$ and **momentum** by $B_j = \frac{D_j - M_j}{2i}$ on $\mathcal{F}^2(\mathbb{C}^n)$, [D'A19]. (If one considers operators on a Hilbert space as generalized complex numbers, then the *adjoint of an operator* T notated as T^* plays the role of the complex conjugate of a complex number.)

In this REU project, students will work with the *creation* operator M and *annihilator* operator D on the complex plane. Students will initiate their investigation from the space of square-integrable complex analytic functions on the unit disk $\mathbb{D}$.

Let $h : \mathbb{D} \to \mathbb{C}$ is a complex analytic function. Then, the area of the image of h, notated as $A(h(\mathbb{D}))$, with multiplicity counted [Ahl78], is defined by $\int_{\mathbb{D}} |Dh(z)|^2 \, dxdy$, the L^2-norm of the derivative of h on $\mathbb{D}$, see [GK06, D'A19]. On the unit disk, $\mathbb{D}$, the excess area generated by the multiplication operator $M = z$ (or area difference), $A(Mh(\mathbb{D})) - A(h(\mathbb{D}))$, is precisely calculated as the average value of the module-square of h on the unit circle times π, [D'A19]. The project aims to generalize the excess area 'equality' with respect to several standpoints, such as the multiplier M, the functional space for h, and the domain $\mathbb{D}$. For some generalizations and further suggestions, see, for example, [BÇGH22].

References

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