Experimental challenges in nuclear astrophysics
Answering to ancient questions

Rosario Gianluca Pizzone

Nature triggers men’s admirations; and we look at everything and wonder, but seldom we investigate the causes; thus we ignore the Movements of the Sun and stars. As well as the explanations of many other phenomena.

Cicero, I century BC
Part 1: Introduction
Observation and understanding of the stars started together with mankind (Denderah Zodiac)
And much progress was made in the last centuries through astronomical studies.

But... it was realized that it was not enough.

In order to understand astrophysical processes, we need to know what’s going on there.
Astrophysics: studying the Universe through the laws of physics

Nuclear Astrophysics: study of nuclear processes which take place in the Universe
Understanding MACROCOSMOS through MICROCOSMOS

WHY?

• to understand how stars produce the energy they emit;
• to understand how chemical elements were produced
• to understand the first seconds of the Universe and help to track how it will end

Why gold costs much more than iron??
Stars emit energy throughout their lives and stars also change (evolve) during their lives. Are these aspects connected? How?

The birth of a start: Galactic gas and powder

Small Mass Star (Sun)  Massive Star
We know from geology Earth is $4.65 \times 10^9$ years old. What source can guarantee solar luminosity for such a long time?

**Gravitational contraction?**
It can be shown Sun can hold from GC for $10^7$ year (Kelvin Helmoltz timescale)

**Nuclear fusion?**
Simple estimates show it’s the right answer. But HOW?

First ideas suggested 4 H nuclei can merge into a He Producing energy from mass defect (Eddington)
Where are the 92 natural elements coming from? How were they produced?

Man: H, C, N, O

Sun: H, He

Earth: Fe, Si, O, Mg

A "cosmic abundance"?
The elemental abundance in the universe is determined in the Solar neighborhood and is assumed to be Universal. It is measured in Earth, Sun, Meteorites, Stars ... by different methods. Several features are visible in the curve of abundance.
Elemental Abundance in the Universe

Elemental abundance in the Universe

![Graph showing elemental abundance](image)

**Features:**

- Li, Be, B under-abundant
- peak around A=56 (Fe)
- almost flat distribution beyond Fe
- exponential decrease up to iron peak
• Eddington 1920, Bethe 1938, von Weiszäcker 1938, Gamow 1948, Cameron 1957 …

In 1957, B²FH presented the basis of the modern nuclear astrophysics in their review paper explaining *nuclear reactions occurring in the interior of the stars*:
- The production of energy
- The creation of elements

**Synthesis of the Elements in Stars**

E. Margaret Burbidge, G. R. Burbidge, William A. Fowler, and F. Hoyle

The first complete review of nuclear reactions explaining: H and He quiescent and hot burning, and of the nucleosynthesis beyond Fe.
Steady stellar burning (P-p chain, CNO cycles, He & C-O burning)

Si - melting & SN

Big Bang Nucleosynthesis

P process

S process

R process

Fe, Ni peak
In the astrophysical environments the energy required for particle interactions is taken from Thermal Energy.

- In the Sun $T=1.5\times10^7$ K then $E=kT\sim$ keV
- In large masses stars $T\sim 10^9$ $E\sim 0.5$–$1$ MeV
Part 2:
Useful definitions
The main problem in the charged particle cross section measurements at astrophysical energies is the presence of the Coulomb barrier between the interacting nuclei. Reactions occur through the **tunnel effect**.

\[
E_{\text{coul}} \sim Z_1 Z_2 \text{ (MeV)}
\]

\[
E_{\text{kin}} \sim kT \text{ (keV)}
\]

\[
V_r \text{ (nuclear well)}
\]

\[
r_0 \text{ (tunnel effect)}
\]
It determines exponential drop in abundance curve!

\[ \text{tunneling probability} \]

\[ P \propto \exp(-2\pi \eta) \]

\[ 2\pi \eta = \text{GAMOW factor} \]

in numerical units:

\[ 2\pi \eta = 31.29 \, Z_1 Z_2 (\mu/E)^{1/2} \]

\( \mu \) in amu and \( E_{cm} \) in keV
Consider reaction \( 1 + 2 \rightarrow 3 + 4 \) \( Q_{12} > 0 \)

Reaction per unit time per unit volume: \( \nu \sigma(v) N_1 N_2 \)

In stellar plasma: \( \varphi(v) \propto \exp\left(-\frac{\mu v^2}{2kT}\right) = \exp\left(-\frac{E}{kT}\right) \)

\( \mu = \text{reduced mass} \)
\( v = \text{relative velocity} \)
\( T = \text{plasma temperature} \)

\( \varphi(v) \) is the Maxwell-Boltzmann distribution for a non-relativistic, non-degenerate gas in thermodynamic equilibrium.

**THEN averaging over \( v \) distribution**

\[
\langle \sigma v \rangle_{12} = \left( \frac{8}{\pi \mu_{12}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) \exp\left(-\frac{E}{kT}\right) E \, dE
\]

Total reaction rate \( R_{12} = (1+\delta_{12})^{-1} n_1 n_2 \langle \sigma v \rangle_{12} \) reactions cm\(^{-3}\) s\(^{-1}\)

\( n_i = \text{number density} \)

\( \langle \sigma v \rangle = \text{KEY quantity to be determined from experiments} \)

\( \Rightarrow \text{NEED ANALYTICAL EXPRESSION FOR } \sigma! \)
The probability for penetrating the Coulomb barrier goes down rapidly with decreasing energy, but at a given temperature the possibility of having a particle of high energy (and therefore high velocity) decreases rapidly with increasing energy (the red curve).

The sum of these opposing effects produces an energy window for the nuclear reaction: only if the particles have energies approximately in this window can the reaction take place.

\[ E_{pp} \sim 20 \text{ keV} \]
\[ E_{SN} \sim 300-800 \text{ keV} \]
\[ E_{BBN} \sim 100-600 \text{ keV} \]
\[ E_0 = f(Z_1, Z_2, T) \]

Most favourable energy region varies with reaction and/or temperature.

**Examples:** \( T \sim 15 \times 10^6 \text{ K} \) \((T_6 = 15)\)

<table>
<thead>
<tr>
<th>reaction</th>
<th>Coulomb Barrier (MeV)</th>
<th>( E_0 ) (keV)</th>
<th>( \Delta E_0 \exp(-3E_0/kT) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p + p )</td>
<td>0.55</td>
<td>5.9</td>
<td>( 7.0 \times 10^{-6} )</td>
</tr>
<tr>
<td>( \alpha + ^{12}\text{C} )</td>
<td>3.43</td>
<td>56</td>
<td>( 5.9 \times 10^{-56} )</td>
</tr>
<tr>
<td>( ^{16}\text{O} + ^{16}\text{O} )</td>
<td>14.07</td>
<td>237</td>
<td>( 2.5 \times 10^{-237} )</td>
</tr>
</tbody>
</table>

Strong sensitivity to Coulomb barrier.

Well-defined stages:
- He-burning
- C/O-burning ...

Area of Gamow peak \( \sim <\sigma v> \) (height \times width)
Part 3:
Direct Methods
in the range nano-picobarn

- severely hindered (1 ev/month)
- and in some cases even beyond present technical possibilities.

Possible solutions: underground measurements, extrapolations

Direct Measurement: Perform the experiment with beam-target interacting at astrophysical energies
Experimental procedure  Often cross sections are too low to be measured

Bare Nucleus Astrophysical $S(E)$-factor is introduced for a easier extrapolation.
measurements performed at higher energies

\[
\sigma_{\text{nr}}(E) = \frac{1}{E} \exp(-2\pi \eta) S(E)
\]
The **DANGER OF EXTRAPOLATION** ...

large uncertainties in the extrapolation!

Necessary is Maximize the signal-to-noise ratio

**SOLUTIONS**

- **IMPROVEMENTS TO INCREASE**
  
  NUMBER OF DETECTED PARTICLES

  $4\pi$ detectors

  New accelerator at high beam intensity

- **IMPROVEMENTS TO REDUCE**
  
  THE BACKGROUND

  Use of laboratory with natural shield - (underground physics)

  Use of magnetic apparatus (Recoil Mass Separator)
Luna underground facility INFN LNGS

TECSA array

TAMU C.S. & INFN LNS
Hard Work is necessary

To understand what we see

To try to go inside the problem
“Some people are so crazy that they actually venture into deep mines to observe the stars in the sky”

*Naturalis Historia - Plinius, 44 A.D.*

**LUNA (Laboratory Underground for Nuclear Astrophysics)**

50 kV accelerator @ Gran Sasso – Italy (1400 m rock -> 10^6 shielding factor)

Two reactions (solar pp chain) already studied at *Gamow peak*:

\[
^3\text{He}^{(3}\text{He},2p)^4\text{He}
\]


\[
d(p,\gamma)^3\text{He}
\]


At lowest energy: \( \sigma \approx 20 \text{ fb} \) \( \rightarrow \) 1 event/month

At lowest energy: \( \sigma \approx 9 \text{ pb} \) \( \rightarrow \) 50 counts/day
The electron screening effect must be taken into account at such low energies.

\[
S_{Sh} \propto S_b \cdot \frac{\pi \eta U_e}{E}
\]

In the accurate measurements for the determination of nuclear cross-sections at the Gamow energy, in laboratory, enhancement \( f_{\text{lab}}(E) \) factor in the astrophysical \( S_b(E) \)-factor has been found.

\[
^3\text{He} + ^2\text{H} \rightarrow p + ^4\text{He}
\]
Electron Screening

At astrophysical energies the presence of electron clouds must be taken into account in laboratory experiments.

The atomic electron cloud surrounding the nucleus acts as a screening potential $U_e$

- Phenomenological approach

$$U_e = \frac{Z_1 Z_2 e^2}{R_a}$$

An experimental measurement of $U_e$ allows:

- a determination of $S_b$ (applications)
- to study electron screening in laboratory conditions and then in stellar plasma
Since direct measurement are extremely time consuming and difficult (at astrophysical energies) or sometimes beyond present possibilities

Independent measurements of cross sections and electron screening potential $U_e$ are needed !!!

We need to be CLEVER: NEW IDEAS ARE NECESSARY

-to measure cross sections at never reached energies

-to retrieve information on electron screening effect when ultra-low energy measurements are available.

INDIRECT METHODS ARE NEEDED
Indirect Methods in Nuclear Astrophysics (both stable and instable beams)

- Coulomb Dissociation
- ANC & transfer reactions
- Trojan Horse Method
- Break-up of loosely bound nuclei
- $\beta$-decay, resonant elastic scattering ...
Part 4: Trojan Horse Method
Trojan Horse Method

Quasi-Free mechanism

Basic idea:

- The B nucleus presents a strong cluster structure: A = x ⊕ S clusters.
- The x cluster (participant) interacts with the nucleus B
- The S cluster acts as a spectator (it doesn't take part to the reaction) and retains the same momentum it had in the entrance channel.

It is possible to extract astrophysically the relevant two-body cross section \( \sigma \) from quasi-free contribution of an appropriate three-body reaction.
We can extract astrophysically relevant two-body cross section $\sigma$

$$B + x \rightarrow C + D$$

from quasi-free contribution of an appropriate three-body reaction

$$A + B \rightarrow C + D + S$$

Coulomb Barrier Suppression

Once Coulomb barrier is overcome by TH nucleus the astrophysical reaction can take place without any evident suppression
Nuclear astrophysics experiments are fun because you never know what you’re going to have as a result…

And like gambling
You hardly have money to cover your expenses

But sometimes you win..
And you get results
In Plane Wave Impulse Approximation (PWIA) the cross section of the three body reaction can be factorized into two terms corresponding to the two vertices.

The cross section for Quasi-Free mechanism PWIA is given by:

$$\frac{d^3\sigma}{dE_c \, d\Omega_c \, d\Omega_D} \propto KF \, \left[ \Phi(q)_{xs} \right]^2 \left( \frac{d\sigma}{d\Omega} \right)_{x + B \rightarrow C + D}$$

where $K_F$ is the kinematical factor.

$|\Phi(q_{xs})|^2$ describes the intercluster $(x-S)$ momentum distribution.

$(d\sigma/d\Omega)$ is the two-body cross section of the virtual reaction $x + B \rightarrow C + D$. 

**Diagram:**

- First vertex: virtual reaction $x + B \rightarrow C + D$
- Second vertex: virtual decay of nucleus $A \rightarrow x + S$
Advantages: Simple & cheap Experimental setup

THM: study of the $^7\text{Li}(p,\alpha)^4\text{He}$ reaction from the 3-body one:

$^2\text{H}(^7\text{Li},\alpha\alpha)n$

TH nucleus deuteron, $E_{\text{beam}} = 21 \text{ MeV}$ @ LNS Catania

Beam energy much higher than Barrier

Angles were selected in such a way that the yield from (the probable) quasi-free mechanism is maximum

Beams and Targets cheap. Detectors set-up trivial

Good ideas make research possible in tough times!!
If one assumes that THM gives the bare nucleus $S$ factor (according to its properties) then by comparing it with direct data one can get the electron screening potential.

\[
S_{Sh} \propto S_b \cdot \frac{\pi \eta U_e}{E}
\]

$U_e = 340 \pm 50$ eV
$U_{ad} = 186$ eV
$S_0 = 16.9$ MeV b

- No screening effect at $E < 100$ keV for indirect data;
- Direct and indirect methods are complementary;
- Independent determination of $S_b(E)$ and $U_e$;
- Previous extrapolations of $S_b$ are confirmed.


$^6\text{Li} + d \rightarrow \alpha + \alpha$
\[
\langle \sigma v \rangle_{12} = \left( \frac{8}{\pi \mu_{12}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) \exp \left( -\frac{E}{kT} \right) E \, dE
\]

Reaction rate obtained for the \(^7\text{Li}(p,\alpha)^4\text{He}\) from THM measure compared with other compilations

Coll. R.G.P, R. Spartà & C.B.
Part 5: $^6,^7\text{Li}$ and its astrophysical relevance
Lithium is important for:

- Probing stellar interiors and structure (need of abundances measurements, stellar modeling, Astroseismology)
- Probing Primordial nucleosynthesis and early universe
- Fusion reactors and electron screening application
Astrophysical applications

Lithium surface abundance for the Sun, Good agreement with NACRE results

RPG et al., A&A 2003
Lithium Destruction in disk stars: astrophysical uncertainties vs. nuclear inputs

Solid lines: THM uncertainties for nuclear rates
Dashed lines: Astrophysical uncertainties (mass=0.9–1 $M_\odot$, He abundance =0.24–0.27, convection efficiency)
The Collaboration


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Further reading

**BOOKS**

- C. Bertulani: *Nuclear Physics in a Nutshell*, Princeton Univ. Press
- C. Iliadis: *Nuclear Physics of Stars* - Wiley

**REVIEW PAPERS**


*Not an exhaustive list!!*
The main difficulties for experimental measurement of this cross section derive from:

• $^{26}\text{Al}$ is an unstable isotope. Moreover also cross sections of reactions induced by metastable state should be known with good precision
• Necessity of a n beam at astrophysical energies
Necessity of a THM measurement

Quasi-free break-up of deuteron

Beam Energy around 60 MeV

Coincidence detection of p and $^{26}$Mg.

This will allow to measure the excitation function of the reaction of interest

In the astrophysical energy range

(0-1 MeV)

Once the 3-particle in exit channel reaction cross section is measured, one can

Extract the binary cross section at astrophysical energies according to the prescriptions of the THM
Resonant reactions

1. Narrow resonances $\Gamma_R \ll E_R$

Breit-Wigner formula

$$\sigma(E)_{BW} = \pi \hbar^2 (1 + \delta_{12}) \frac{2J + 1}{(2J_1 + 1)(2J_2 + 1)} \frac{\Gamma_a \Gamma_b}{(E - E_r)^2 + (\Gamma/2)^2}$$

Insert in expression for reaction rate, integrate and get:

$$\langle \sigma v \rangle_{12} = \left( \frac{2\pi}{\mu_{12} kT} \right)^{3/2} \hbar^2 (\omega \gamma) \exp \left( -\frac{E_R}{kT} \right)$$

(for single resonance)

$$\langle \sigma v \rangle_{12} = \left( \frac{2\pi}{\mu_{12} kT} \right)^{3/2} \sum_i (\omega \gamma)_i \exp \left( -\frac{E_{R_i}}{kT} \right)$$

(for many resonances)

$$(\omega \gamma)_R = (1 + \delta_{12}) \frac{2J + 1}{(2J_1 + 1)(2J_2 + 1)} \frac{\Gamma_a \Gamma_b}{\Gamma}$$

resonance strength

$\rightarrow$ integrated cross section over resonant region

Experiment: determine $(\omega \gamma)_R$ and $E_R$

N.B. $\langle \sigma v \rangle_{12} \propto \exp \left( -\frac{E_R}{kT} \right)$

Low-energy resonances ($E_R \rightarrow kT$) are VERY important
2. Broad resonances $\Gamma_R \sim E_R$

Breit-Wigner formula

energy dependence of partial and total widths

N.B. Overlapping broad resonances of same $J^\pi$ → interference effects

3. Sub-threshold resonances

$S$-factor can be entirely dominated by contribution of sub-threshold state(s)

if interference effects are negligible, total reaction rate

$$<\sigma v>_{tot} = <\sigma v>_r + <\sigma v>_{nr}$$
However, the electron screening effect must be taken into account.


In the accurate measurements for the determination of nuclear cross-sections at the Gamow energy, in laboratory, enhancement $f_{\text{lab}}(E)$ -factor in the astrophysical $S_b(E)$-factor has been found:

$$S_{Sh} \propto S_b \cdot \frac{\pi \eta U_e}{E}$$

$$\begin{array}{c}
\text{E (KeV)} \\
\text{5.5} \quad \text{6.5} \quad \text{7.5} \quad \text{8.5} \quad \text{9.5}
\end{array}$$

$$\begin{array}{c}
\text{S (MeV-b)} \\
\text{5.5} \quad \text{6.5} \quad \text{7.5} \quad \text{8.5} \quad \text{9.5}
\end{array}$$

$$\begin{array}{c}
\text{3He} + \text{2H} \rightarrow \text{p} + \text{4He}
\end{array}$$
$U_e = 340 \pm 50$ eV
$U_{ad} = 186$ eV
$S_0 = 16.9$ MeV b


- No screening effect at $E < 100$ keV for indirect data;
- Direct and indirect methods are complementary;
- Independent determination of $S_b(E)$ and $U_e$;
- Previous extrapolations of $S_b$ are confirmed.
Data Analysis Phases:

- Find the 3-body reaction of interest among the ones occurring in the target.
- Separate the quasi-free mechanism from all the others
- Measure the binary reaction cross section from the three body one
- Normalization and comparison to direct data: validity test and measurement of astrophysical interest
- Extraction of electron screening potential, reaction rate and so on.
Two body reaction takes place at:

\[ E_{qf} = E_{Bx} - B_{x-S} = E_{cD} - Q_{2b} \]

Where

- \( E_{Bx} \) is the beam energy in the center of mass of the two body reaction.
- \( B_{x-S} \) binding energy of the two clusters inside the Trojan Horse plays a key role in compensating for the beam energy (under proper kinematical conditions).

\[ E_{qf} \approx 0 \]
In Plane Wave Impulse Approximation (PWIA) the cross section of the three body reaction can be factorized into two terms corresponding to the two vertices.

\[
\frac{d^3\sigma}{dE_c \, d\Omega_c \, d\Omega_D} \propto KF \left[ |\Phi(q_{xs})|^2 \right] \left( \frac{d\sigma}{d\Omega} \right)
\]

KF \hspace{1cm} \text{kinematical factor}

\[ |\Phi(q_{xs})|^2 \] describes the intercluster (x-S) momentum distribution. 

\[ \frac{d\sigma}{d\Omega} \] two-body cross section of the virtual reaction \( x + B \rightarrow C + D \).